Essays on Measuring, Modeling and Forecasting Time-varying Risk in Financial Markets

by

Xuan Xie

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Abstract

This thesis studies four related topics in financial economics; realized volatility modelling and forecasting in the presence of model instability, forecasting stock return realized volatility at the quarterly frequency, quarterly realized beta measurement and beta neutrality evaluation under a popular long short strategy.

Recent advances in financial econometrics have allowed for the construction of efficient *ex post* measures of daily volatility. The first topic investigates the importance of instability in models of realized volatility and their corresponding forecasts. Testing for model instability is conducted with a subsampling method. We show that removing structurally unstable data of a short duration has a negligible impact on the accuracy of conditional mean forecasts of volatility. In contrast, it does provide a substantial improvement in a model's forecast density of volatility. In addition, the forecasting performance improves, often dramatically, when we evaluate models on structurally stable data.

The second topic is on forecasting stock return volatility at quarterly level. The last decade has seen substantial advances in the measurement, modeling and forecasting of volatility which has centered around the realized volatility literature. To date, most of the focus has been on the daily and monthly frequency, with little attention on longer horizons such as the quarterly frequency. In finance applications, forecasts of volatility at horizons such as quarterly, are of fundamental importance to asset pricing and risk management. In this chapter we evaluate models for stock return volatility forecasting at the quarterly frequency. We find that an autoregressive model with one lag of quarterly

realized volatility produces the most accurate forecasts, and dominates other approaches, such as the recently proposed mixed-data sampling (MIDAS) approach.

Chen and Reeves (2009) introduced a new beta measurement technique via the Hodrick-Prescott filter and found it substantially reduced measurement error and produced much better performance than Fama-MacBeth measurement approach at the monthly frequency. The third chapter extends this technique to quarterly beta measurement. The finding in Chen and Reeves (2009) is also confirmed at the quarterly frequency. Hodrick-Prescott filtered beta contains the most relevant information and follows closely the true underlying beta. This result is also used in the final chapter to construct the proxy for the true underlying quarterly beta time series.

The final topic is to investigate the economic value of realized beta. Market neutral funds are commonly advertised as alternative investments offering returns which are uncorrelated with the broad market. Utilizing recent advances in financial econometrics we demonstrate that constructing market neutral funds from monthly return data can be widely inaccurate. Given the monthly frequency is the most common for return measurement in the hedge fund industry, our findings highlight the need for higher frequency return data to be more commonly utilized. We demonstrate the use of daily returns to achieve a more market neutral portfolio, relative to the case of only using monthly returns.

Co-Authorship

Chapter 2 is co-authored with Dr. Jonathan Reeves, UNSW and Associate Professor John Maheu, University of Toronto, Canada. Chapter 3 is co-authored with Dr. Jonathan Reeves, UNSW. Chapter 5 is co-authored with Dr. Jonathan Reeves, UNSW and Associate Professor Nicolas Papageorgiou, HEC, University of Montreal, Canada.

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Chapter 1

Introduction

There has been tremendous progress in realized variance and co-variance theory in recent years, coupled with significant advances in technology which makes computing power faster and cheaper. These econometric techniques have spurred widespread interest in both academic research and private industry applications. The focus of this thesis is on two related topics: realized volatility and realized beta.

Chapter two and three extend the empiricial research on realized volatility. A series of works in Andersen et al. (2001a), Andersen et al. (2001b) and Andersen et al. (2003) have popularized the concept of realized volatility. Realized volatility generally provides an estimate of integrated volatility plus jump components. The realized bi-power by Barndorff-Nielsen and Shephard (2004b) offers a way to separate two components of quadratic variation. Chapter two studies the structural stability for both realized volatility and realized bi-power by Andrews (2003). Furthermore, the presence of breaks in the volatility time series is considered when forecasting realized volatility. A method is proposed to remove structurally unstable data. The forecast performance and forecast density are evaluated against models with full sample data. While chapter two evaluates the impact of structure break on foregin exchange volatility forecasting, chapter three compares the stock price volatility forecasting performances of several competing models. A substantial body of research has been directed to volatility forecasting at daily, weekly or monthly frequencies, however, quarterly volatility is of great interest to market participants as well. Practitioners need accurate volatility forecasting to construct volatility curve, especially with maturities of one week, one month and one quarter. Chapter three contributes to the literature on stock return volatility forecasting performance evaluation at the quarterly frequency and evaluates three classes of realized volatility forecasting models. The first class is autoregressive models with various lags and varying in-sample sizes. It is simple but often delivers the superior forecasting performance as found in Andersen et al. (2003) for short horizon volatility forecasts. Constant models have been the industry standard due to its simplicity. The forecasting ability of constant models with various in-sample estimation periods are studied. Mixed-Data Sampling (MIDAS) approachs, proposed by Ghysels et al. (2005 and 2006), have been found to provide some improvements in short horizon volatility predictions. Furthermore, Ghysels et al. (2009) shows MIDAS approach offers superior forecasting performance, compared to popular models such as GARCH. Chapter three evaluates the MIDAS approach, relative to other time series approaches.

Chapter four and five contribute to the realized beta literature. The theoretical work for the construction of the realized beta is established by Barndorff-Nielsen and Shephard (2004b) and Andersen et al. (2006). They documented that realized beta is persistent and predictable and Hooper et al. (2008) illustrates that a simple autoregressive model can produce accurate beta forecasts. The recent beta measurement technique, proposed in Chen and Reeves (2009), has significantly reduced the measurement error in monthly beta estimation. Chapter four extends this technique to the quarterly frequency and confirms the usefulness to investors who do not have access to high-quality high-frequency price data. This technique enables investors to better evaluate a firm's systematic risk with freely available daily price data. Furthermore, this technique is used in chapter five to construct the proxy of true quarterly realized beta time series over a long history. Market neutral strategies are some of the most popular investment approaches used in the alternative investment industry. However, the recent financial crisis reveals that many of them have substantial correlation with the market. The final chapter replicates a popular investment strategy by constructing a momentum-based market neutral equity portfolio comprising the S&P 100 Index, and demonstrates the impact of the beta estimation approach on the ex-post beta exposure of the portfolio. The recent advances in realized beta construction and measurement techniques utilizing daily data are shown to produce a better assessment of market risk exposure.

This thesis is organised as follows. Chapter two is on the realized volatility modelling and forecasting in the presence of model instability. Chapter three is on forecasting stock return volatility at quarterly frequency. Chapter four is on quarterly beta measurement. Chapter five presents an evaluation of quarterly beta forecasting techniques in a market neutral setting. Chapter six concludes.

Chapter 2

Forecasting Volatility in the Presence of Model Instability

2.1 Introduction

Characteristics of the second moment of asset returns, such as predictability and distributional features, play a major role in the implementation of portfolio choice, risk management and asset pricing. Recent advances in financial econometrics have allowed for the construction of efficient *ex post* measures of daily volatility. These nonparametric volatility measures, often called *realized volatility*, permit the direct modeling of volatility dynamics using observable data. The justification of these improved measures of volatility was introduced by Andersen and Bollerslev (1998) in the context of measuring the performance of GARCH (Engle (1982) and Bollerslev (1986)) model forecasts.

Andersen et al. (2001b, 2003), and Barndorff-Nielsen and Shephard (2002) reviewed the theory of quadratic variation for continuous time semi-martingale processes and the realized volatility estimator. In the absence of market microstructure effects, the sum of intraday squared returns provides a consistent estimate of integrated volatility plus squared jump increments. This estimator, which we call realized volatility (also referred to as 'realized variance') provides a much more efficient estimate of *ex post* volatility than traditional measures such as GARCH estimators.

As mentioned, realized volatility in general provides an estimate of integrated volatility plus any jump increments. Barndorff-Nielsen and Shephard (2004a) showed that the jump component and integrated volatility can be consistently estimated separately. By using a *realized bi-power* estimate of integrated volatility, it is possible to separate the two components of quadratic variation. (Another useful measure of *ex post* volatility is power variation, which includes quadratic variation as a special case. See Barndorff-Nielsen and Shephard (2004a), for details.) This permits the study of the statistical properties of integrated volatility and the jump component in quadratic variation.

The widespread availability of high frequency intraday data has popularized the use of realized volatility as observable data on volatility. This has led to the building of traditional time-series models for the purpose of studying the dynamics and forecasting performance of volatility. Examples include Andersen et al. (2001a), Andersen et al. (2003), Andersen et al. (2007), Maheu and McCurdy (2002), Koopman et al. (2005), Ghysels et al. (2006), and Martens et al. (2009). Andersen et al. (2005) studied the loss in forecast precision from measurement error when using realized volatility.

One characteristic of FX time series is the presence of structural breaks, which typically occur around the important data releases, unusual event and major policy change annoucement. Their impact on forecasting volatiliy is unclear. The purpose of this chapter is to investigate the presence of structural instability in simple reduced form models of realized volatility and their effects on model forecasts. Based on parametric forecasting models, we investigate model stability in both realized volatility and realized bi-power variation. Our statistical approach is based on the end-of-sample instability tests of Andrews (2003), which are designed to provide good statistical performance when the period of the sample being tested for instability may be very small relative to the rest of the data. For Bayesian approaches, see Barnett et al. (1996) and Gerlach et al. (2000).

We show that the tests have good size characteristics for the models and sample sizes we consider. A second contribution of the chapter is to propose a method to adjust forecasts when breaks in the data generating process (DGP) have been identified. We study the benefits of removing blocks of data identified as breaks and whether this improves forecasting performance. Intuitively, a break is a change in the parameters and/or error distribution of the DGP. Formally, a block of data is identified as a break when the Andrews (2003) end-of-sample instability test is significant.

Our empirical investigation focuses on daily foreign exchange volatility for the JPY-USD and DEM-USD markets. Using several autoregressive models as well as different block sizes to identify breaks, we find clear evidence of model instability of short duration in the volatility of both foreign exchange markets.

A natural question is whether the breaks are associated with jumps in the return process. To investigate this, we first measure the correlation between breaks and the jump component estimate which is approximately 0.34. To investigate this further, we apply break tests to time-series models of bi-power variation. Note that bi-power is a consistent estimate of integrated volatility and by assumption does not contain discrete jumps. We find strong evidence of breaks in this series and a high correlation of breaks found in realized volatility and bi-power variation. Our conclusion is that the source of the majority of breaks is from integrated volatility. (In general we cannot rule out jumps in the volatility process as the cause of breaks.)

The overwhelming evidence of breaks suggests that ignoring them may bias forecasts. That is, better forecasting results may be possible by removing structurally unstable data from estimation. We investigate this by removing data that has been identified as a break in model estimation. First, we show that removing structurally unstable in-sample data (model estimation data) of a short duration, has a negligible impact on the accuracy of conditional mean forecasts of volatility. In contrast, it does provide a substantial improvement in a model's forecast density of volatility. In addition, the forecasting performance improves when we evaluate models on structurally stable out-of-sample data. That is, models' forecasting performance improves, often dramatically, when out-of-sample break data are removed from the evaluation sample.

This chapter is organized as follows. The next section reviews the Andrews test for structural instability applied to the linear regression model. Section 2.3 discusses the estimation of *ex post* volatility measures from high frequency intraday data. Details on the data sources are in Section 2.4, while the identification of breaks and how they affect out-of-sample point and density forecasts are found in Section 2.5. Section 2.6 concludes the chapter.

2.2 Testing for Structural Instability

In this section we give a brief review of the testing method of Andrews (2003). In the following empirical investigation, all our models can be cast into a linear regression model and therefore we consider the identification of breaks in the regression model,

$$Y_t = X_t\beta + \epsilon_t, \tag{2.1}$$

where X_t has d regressors, Y_t is a scalar, and $\mathrm{E}(\epsilon_t X_t) = 0$. For this specification, the test for instability of short duration is very general and admits nonnormal innovations, conditional heteroskedasticity, and long memory in the observations and/or innovations. The main requirement, under the null hypothesis of no breaks, is that the data is strictly stationary and ergodic.

Consider the following end-of-sample break model:

$$\begin{aligned} Y_t &= X_t\beta + \epsilon_t, && t = 1, ..., T-m, && (2.2)\\ Y_t &= X_t\beta_t + \epsilon_t, && t = T-m+1, ..., T. && (2.3) \end{aligned}$$

The null hypothesis is '$\beta_t = \beta$ for $t = T-m+1, ..., T$, and $\{Y_t, X_t\}_{t=1}^{\infty}$ is stationary and ergodic' against an alternative of '$\beta_t \neq \beta$ for some $t = T-m+1, ..., T$ and/or the distribution of $\{\epsilon_t\}_{t=T-m+1}^{T}$ differs from $\{\epsilon_t\}_{t=1}^{T-m}$.' The choice of the break length m is chosen by the econometrician, and Andrews showed that two cases arise. In the following, let the subscript $i:j$ denote observations i through to j in a vector or matrix. Define the S test when $m \geq d$ as $S = S_{T-m+1}(\hat{\beta}_T, \hat{\Omega}_T)$, where

$$\begin{aligned} S_j(\beta, \Omega) &= A_j(\beta, \Omega)^T V_j^{-1}(\Omega) A_j(\beta, \Omega), && (2.4)\\ A_j(\beta, \Omega) &= X_{j:j+m-1}^T \Omega^{-1}(Y_{j:j+m-1} - X_{j:j+m-1}\beta), && (2.5)\\ V_j(\Omega) &= X_{j:j+m-1}^T \Omega^{-1} X_{j:j+m-1}, && (2.6) \end{aligned}$$

$\hat{\beta}_T$ is the Ordinary Least Squares (OLS) estimate using all data ($t = 1, ..., T$), and the $m \times m$ covariance matrix is estimated by

$$\hat{\Omega}_T = \frac{1}{T-m+1} \sum_{j=1}^{T-m+1} (Y_{j:j+m-1} - X_{j:j+m-1}\hat{\beta}_T)(Y_{j:j+m-1} - X_{j:j+m-1}\hat{\beta}_T)^T. \quad (2.7)$$

When $m < d$, the test statistic has the form $S = P_{T-m+1}(\hat{\beta}_T, \hat{\Omega}_T)$, where

$$P_j(\beta, \Omega) \;\; = \;\; (Y_{j:j+m-1} - X_{j:j+m-1}\beta)^T \Omega^{-1} (Y_{j:j+m-1} - X_{j:j+m-1}\beta). \quad (2.8)$$

In this test, m is fixed as $T \to \infty$, and therefore the test is not consistent. Instead Andrews showed that the test is asymptotically unbiased, and a subsampling procedure can be used to obtain a p-value. It is obtained as

$$\text{p-value} = \frac{1}{T-2m+1} \sum_{j=1}^{T-2m+1} I(S \leq S_j), \quad (2.9)$$

where $I(\cdot) = 1$ when the argument is true, and 0 otherwise. Note that, in this calculation,

$$S_j = S_j(\hat{\beta}_{2(j)}, \hat{\Omega}_T), \;\; j = 1, ..., T-2m+1, \quad \text{if} \;\; m \geq d \quad (2.10)$$

$$S_j = P_j(\hat{\beta}_{2(j)}, \hat{\Omega}_T), \;\; j = 1, ..., T-2m+1, \quad \text{if} \quad m < d \quad (2.11)$$

where $\hat{\beta}_{2(j)}$ is the OLS estimate with observations $t = 1, ..., T-m$, and $t \neq j, ..., j + \lceil m/2 \rceil - 1$, where $\lceil m/2 \rceil$ is the smallest integer greater than or equal to $m/2$.

2.3 Measuring Volatility

To illustrate our approach to volatility measurement, consider the following class of continuous-time jump diffusions used in Andersen et al. (2003), in which the logarithmic price process $\{p(t)\}_{t\geq 0}$ follows the model

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + J(t)dq(t). \tag{2.12}$$

Here $W(t)$ is standard Brownian motion, $\sigma(t)$ is the volatility process, $\mu(t)$ has bounded and finite variation, $J(t)$ is the jump size, and $q(t)$ is a counting process such that $dq(t) = 0$ when there is no jump and $dq(t) = 1$ when there is a jump. The jump intensity is $\lambda(t)$. The quadratic variation of the increment in prices, or the return $r(t) = p(t) - p(t-1)$, is

$$QV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{t-1<s\leq t} J^2(s). \tag{2.13}$$

Suppose the process is sampled N times per day on an equally-spaced grid. Then define the $\delta = 1/N$ period returns as $r_{t,i} = p(t+i\delta) - p(t+(i-1)\delta)$, $i = 1, 2, ..., N$. Define realized volatility as the sum of squared intraday returns sampled at frequency δ,

$$RV_{t+1} = \sum_{i=1}^{N} r_{t,i}^2. \tag{2.14}$$

In the absence of market microstructure dynamics, RV_t is a consistent estimator of QV_t as $N \to \infty$. For additional details on the class of processes and technical assumptions underlying these estimators see Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004a).

Barndorff-Nielsen and Shephard (2004a) have proposed

$$BP_{t+1} = \frac{\pi}{2} \sum_{i=2}^{N} |r_{t,i-1}||r_{t,i}|, \tag{2.15}$$

called realized bi-power variation, as a consistent estimator (as $N \to \infty$) of integrated volatility,

$$\int_{t}^{t+1} \sigma^2(s)ds. \tag{2.16}$$

This allows for a consistent estimator of the jump component $\sum_{t-1<s\leq t} J^2(s)$ as $RV_t - BP_t$, which is empirically measured as $\max(RV_t - BP_t, 0)$, following Barndorff-Nielsen and Shephard (2004a).

2.4 Data

In the next section we investigate breaks in two time-series of foreign exchange volatility, JPY-USD and DEM-USD. The data for the foreign exchange market were obtained from Olsen Financial Technologies GmbH, Zurich, Switzerland, and is an extension of that used in Maheu and McCurdy (2002). We use returns based on the midpoint of five minute 'bid' and 'ask' quotes from the foreign exchange market to construct daily realized volatility and bipower variation. To minimize market microstructure effects, we filter the five minute return data with an MA(4) specification. We refer the reader to Maheu and McCurdy (2002) for additional details on foreign exchange (FX) data construction, removal of slow trading days, and holidays as well as the filtering. The final data ranges are 1986/12/16 – 2002/12/31 JPY (4001 observations), and 1986/12/11 – 2002/12/31 DEM-USD (4001 observations). The choice of the DEM-USD might have some impact, however it is still one of the most liquid foregin exchanges before the introduction of EUR-USD and it should be valid for this study.

2.5 Results

The positive skew and high kurtosis for all three volatility time series are consistent with the volatility literature. The mean and standard deviation of JPY-USD realized volatility is higher than realized bi-power and also higher than DEM-USD realized volatility, which implies JPY-USD is considerably more volatile than DEM-USD.

2.5.1 Structural Instability in Volatility?

The test for structural instability detailed in Section 2.2 is conditional on a model. Andersen et al. (2001b) showed that the logarithmic transformation of realized volatility makes the distribution more Gaussian. In addition, Andersen et al. (2003) examined a variety of realized volatility models without finding any models that dominated the AR(5) and AR(10) realized volatility models. Therefore, we consider the following specifications of $\log(RV_t)$: (i) an AR(5) model, (ii) an AR(10) model, and (iii) a mixed model of the form

$$\log RV_t = \phi_0 + \sum_{i=1}^{5} \phi_i \log RV_{t-i} + \gamma \log RV_{t-6,t-10} + \epsilon_t, \quad \epsilon_t \sim \mathrm{iid}(0, \sigma^2), \tag{2.17}$$

where $\log RV_{t-6,t-10} = \frac{1}{5}(\log RV_{t-6} + \cdots + \log RV_{t-10})$. This latter specification is motivated by the long-memory model of Corsi (2009). We also include, in some cases, results for exactly the same set of models with RV_t replaced by realized bi-power variation BP_t.

To test a model and data for instability, we use the previous 1000 observations (with no previous observations removed) when applying the Andrews (2003) testing procedure. To let us investigate the statistical properties of the test, Table 2.1 reports the size of an end-of-sample instability test for a typical volatility DGP, both at the 0.05 and 0.01 nominal significance levels. For sample sizes of $T = 100$ and 500, for $m = 1, 5$ and 10, and for different error distributions, the true size is close to nominal. In Monte Carlo simulations, Andrews (2003) demonstrated favourable power properties of this testing procedure. Note that the repeated use of the Andrews test may result in a size distortion.

Figure 2.1 displays realized volatility of the JPY-USD, and breaks (displayed by vertical lines) are identified from an AR(5) model with $m = 1, 5, 10$ and a 0.025 nominal significance level. Most of the breaks identified with $m = 10$ are also identified with a smaller m. Figure 2.2 displays the logarithm of the DEM-USD volatility series along with a break indicator for $m = 1, 5, 10$ and 0.01 nominal significance level, measured from an AR(5) model. There is approximate agreement over models and block sizes as to the episodes that contain breaks.

It may appear that a large proportion of the data is identified as breaks. However, this is not the case, and in fact a very small percent of the data contain breaks. Table 2.3 reports the proportion of days throughout the entire sample, excluding the first 1000 observations, that are found to be a break day. For realized volatility with $m = 1$ and a 0.01 and 0.025 nominal significance level, typically around 0.008 and 0.02 of the days are found to be breaks, respectively.

To investigate the possibility of structural instability of longer duration, the frequency of breaks in the previous 20 days is computed and displayed in Figure 2.4. As this frequency remains low, typically below four over our sample period, we conclude there is no long duration structural instability in our volatility samples. If medium or long duration structural instability was indicated by the frequency exceeding five for a substantial period, dummy variables could potentially be employed in the model estimation for the case of a level shift.

For the case of $m = 1$, a natural question is whether the test is mainly accounting for jumps found in quadratic variation. Recall from Section 2.3, that realized volatility is a consistent estimate of integrated volatility plus any squared jump increments. Generally jumps are large and display different dynamics from the instantaneous volatility process. To investigate this, Figure 2.3 repeats the above procedure for an AR(10) model of bi-power variation for the JPY-USD. There are a number of breaks that appear in close correspondence to the breaks in the realized volatility models. Panel A of Table 2.4 verifies this impression by reporting a high correlation between breaks found in realized volatility and bi-power models. The correlation of the break indicator variable for realized volatility with the break indicator variable for bi-power variation, $m = 1$, is 0.8431 for the AR(10) specification. For the other models, AR(5) and Mixed, the correlation is 0.8438 and 0.8459, respectively. This suggests that the breaks are common to both measures of volatility. Furthermore, as shown in panel B, there is a much lower correlation between breaks in the realized volatility models and the nonparametric jump estimate of Barndorff-Nielsen and Shephard (2004a). The latter jump component of quadratic variation is estimated as $\max(RV_t - BP_t, 0)$. We conclude that there are a number of common structurally unstable observations, which are blocks of observations that are identified as breaks, in realized volatility and bi-power time-series models for our datasets.

2.5.2 Does model instability affect forecasting performance?

In this subsection we investigate if eliminating identified breaks leads to improved out-of-sample forecasting performance. One-step-ahead forecasting of $\log RV_{t+1}$ and RV_{t+1} was conducted for our realized volatility time series, and the forecast evaluation period was from observation 2750 until observation 4001. A forecast of RV_{t+1} was obtained from the forecast of $\log RV_{t+1}$, assuming log-normality. All model estimation was based on data going back to observation 1011 in the dataset. (Results based on models that use a rolling sample of data were very similar.) Two methods were considered. Method *Include* is when breaks are not identified and the forecast is based on all pre-forecast data, starting from observation 1011. Method *Exclude* is when breaks are identified by the Andrews test with $m = 1$, using the previous 1000 observations (with no previous observations removed), and the associated observations were removed from the sample for parameter estimation in forecasting. The out-of-sample forecast is based on the estimated model parameters and the most recent information set, ie. $\mathrm{E}(Y_{T+1}) = X_{T+1}\hat{\beta}$. The information set for the *Include* Method contained all pre-forecast data, whereas the information set for the *Exclude* Method contained all pre-forecast data, with the exception of any break days, which was achieved by removing the day and leading ahead the dataset by one day.

Our discussion will focus on the $m = 1$ case. However, this short block length will have power to detect structural instability of a longer duration by sequentially testing each observation. In contrast, the use of a larger block length may remove some structurally stable observations, since the block length may be longer than the number of unstable observations.

Conditional mean forecasts of volatility were evaluated by three alternative measures,

R^2, mean squared error (MSE) and mean absolute error (MAE). The R^2 is from a regression over the forecast evaluation period of $Y = \beta_0 + \beta_1 X + \epsilon$ where Y is either realized volatility or the logarithm of realized volatility, and X is the respective forecast. Both forecasting methods were considered, Include and Exclude, and Tables 2.5 and 2.6 report these results for the JPY-USD realized volatility series, from the AR(5), AR(10) and Mixed models. Panel A of these tables uses a 0.01 nominal significance level to test for breaks and displays forecasting performance on all out-of-sample data. Panel B displays forecasting performance over the non break data. That is, *ex post* we evaluated the loss functions only on data that was found to contain no evidence of structural instability. This represents the hypothetical improvement we would see if we could focus only on forecasting the structural stable portions of the data according to our testing procedure. Panels C and D repeat the first two panel, but using a nominal significance level of 0.025.

The results suggest that there are no clear benefits to conditional mean forecasts of volatility from removing blocks of unstable data. In several cases, forecast precision is worse when breaks are accounted for. This result is not too surprising, given that the proportion of the sample that is identified as breaks is small, and that the loss functions are averages which tend to minimize the effect of breaks in the data. With sufficiently large samples, an accurate conditional mean forecast of volatility can still be generated when breaks of short duration are included in the estimation sample. Finally, the AR(10) and Mixed models provide marginally better forecasts than the AR(5) model.

On the other hand, panels B and D of the tables show substantial forecast improvements when breaks are (*ex post*) removed from the forecast horizon. For instance, in Table 2.5A and 2.5B the MSE decreases from 0.1894 to 0.1656 for the AR(5) model of the JPY-USD series, when breaks are removed from the data. The improvements are more dramatic in forecasts of RV_{t+1}. Panels A and B of Table 2.6 show the R^2 to increase from about 0.28 to 0.59 for the AR(10) model. This is a doubling in forecasting power when structurally unstable observations are removed. Note that very few structurally unstable observations, less than 1% of the forecast period, are detected, but these observations are extremely influential in affecting our loss functions. For the 1252 out-of-sample observations, we identify 10, 11, and 11 observations based on the AR(5), AR(10), and Mixed models, respectively for the JPY-USD series at a 1% significance level. Similarly, for the DEM-USD series, the numbers of observations identified as unstable are 11, 10, and 10. Similar improvements occur for other model specifications and the DEM-USD series. Thus, the true predicting ability of the models is clearly understated when structurally unstable observations are not removed from the forecast sample.

2.5.3 Does model instability affect the forecast density?

Finally, we consider the effect that breaks may have on the forecast density of RV_{t+1}. Predicting the future distribution of volatility plays a major role in financial risk management, asset pricing and portfolio allocation decisions. One-step-ahead forecasts of the predictive density of volatility are considered. The empirical coverage of the percentiles of the one-step-ahead predictive distribution is measured as the percentage of the days during the forecast evaluation period in which the realized volatility was less than the forecast percentile. Percentiles of the predictive density are based on a Normality assumption on $\log(RV_{t+1})$ in each of the models. (Note that the coverage for the $\log(RV_{t+1})$ forecast density will be identical to coverage for the RV_{t+1} forecast density.) Table 2.7 reports these results for the AR(5), AR(10) and Mixed models at the 0.01 and 0.025 nominal significance levels for the JPY-USD series. Figure 2.5 displays the absolute error in coverage for the AR(10) model for both the JPY-USD and DEM-USD series. Excluding unstable observations results in improvements across the entire distribution.

Overall, the results are similar across our two realized volatility time series. There is a slight upward bias in the 50th percentile estimate when breaks are included in the estimation. For the JPY-USD series, this coverage is 0.5495, 0.5519 and 0.5471 for the AR(5), AR(10) and Mixed models, respectively. When breaks are excluded from the estimation with a 0.025 nominal significance level, the coverage falls to 0.5072, 0.5224 and 0.5120, respectively. Other notable improvements occur at other parts of the distribution. For example, the 75th percentile coverage for the JPY-USD series falls from 0.8035 when including breaks, to 0.7572 when excluding breaks, with a 0.025 nominal significance level, from an AR(10) model. Overall, excluding break data from the estimation with a 0.025 nominal significance level results in a more accurate estimate of the future distribution of volatility. The result is quite useful to the risk manager which is interested in future distribution of volatility and calculating VaR.

2.6 Conclusions

Structure break is common to foreign exchange time series, which might be caused by central bank interventation, other momentary policy or significant market events. These structure breaks normally have short duration. The Andrews (2003) tests enable identification of structural instability of short duration, under very general conditions. We applied this testing approach to the JPY-USD and DEM-USD realized volatility models, and examined the impact on forecasting performance. In general, we found a high correlation between unstable observations in realized volatility and realized bi-power variation. We showed that removing structural unstable observations of short duration has a negligible impact on the accuracy of conditional mean forecasts of volatility. In contrast, it does provide a substantial improvement in a model's forecast density of volatility. In addition, the forecasting performance improved when we evaluated models on structurally stable data. That is, models' forecasting performance improved, often dramatically, when break data was removed from the evaluation sample.

Table 2.1: Descriptive Statistics of daily realzied volatility

	JPY Realized Volatility	JPY Realized Bi-Power	DEM Realized Volatility
Mean	0.5992	0.5306	0.5199
Stdev	0.7916	0.6613	0.4549
Min	0.0290	0.0265	0.0369
Max	34.4000	27.5000	10.9000
Skew	21.6670	19.3444	6.2002
Kurtosis	848.6843	710.7816	89.6972

This table presents the descriptive statistics for daily realized volatiliy of JPY-USD and DEM-USD and realized bi-power of JPY-USD. We show the mean, standard deviation, minimum, maximum, skew and kurtosis. The final data ranges are 1986/12/16 – 2002/12/31 JPY (4001 observations), and 1986/12/11 – 2002/12/31 DEM-USD (4001 observations).

Table 2.2: True Size Estimates

	m	$z_t \sim N(0,1)$ T=100	$z_t \sim N(0,1)$ T=500	$z_t \sim t(6)$ T=100	$z_t \sim t(6)$ T=500	$z_t \sim t(12)$ T=100	$z_t \sim t(12)$ T=500
A							
	10	0.0499	0.0516	0.0608	0.0524	0.0545	0.0507
	5	0.0340	0.0473	0.0402	0.0503	0.0405	0.0487
	1	0.0287	0.0468	0.0324	0.0437	0.0300	0.0463
B							
	10	0.0200	0.0149	0.0350	0.0170	0.0257	0.0144
	5	0.0113	0.0100	0.0171	0.0126	0.0158	0.0104
	1	0.0045	0.0104	0.0060	0.0097	0.0063	0.0106

This table reports size estimates of the S test. Panels A and B record results for 0.05 and 0.01 nominal significance levels, respectively. Data is simulated from the following AR(10) DGP:

$$\begin{aligned} y_t &= X_t\beta + \sigma z_t, \quad t = 1, \ldots, T, \\ X_t &= [1, y_{t-1}, ..., y_{t-10}] \end{aligned}$$

where z_t is either standard normal or a standardized t-innovation, $\beta = (-0.160, 0.467, 0.106, 0.038, 0.073, 0.083, -0.024, 0.020, 0.024, 0.029, 0.018)^\top$, and $\sigma = 0.4995$. The first 5000 draws from the DGP were discarded in forming a sample for each iteration. 10 000 repetitions were used to estimate size. Standard errors are $\sqrt{\hat{p}(1-\hat{p})/N}$, $N = 10\,000$, where $\hat{p}$ is the size estimate.

Table 2.3: Proportion of Days that are Unstable over Full Sample

		AR(5)	AR(10)	Mixed
A				
	JPY-USD $\log(RV_t)$	0.0087	0.0087	0.0094
	DEM-USD $\log(RV_t)$	0.0080	0.0080	0.0074
B				
	JPY-USD $\log(RV_t)$	0.0247	0.0227	0.0241
	DEM-USD $\log(RV_t)$	0.0187	0.0184	0.0197

Panels A and B record results for the 0.01 and 0.025 nominal significance levels, respectively, where breaks are identified by the S test with $m = 1$, based on data going back 1000 observations in the dataset. The proportion is computed over the full sample, excluding the first 1010 observations.

Table 2.4: Correlations between Unstable Observations in $\log(RV_t)$ and $\log(BP_t)$ and Jumps for the JPY-USD

	AR(5)	AR(10)	Mixed
A	0.8438	0.8431	0.8459
B	0.3396	0.3412	0.3465

Panel A reports the correlation of the break indicator variable for the $\log(RV_t)$ model with the break indicator variable for the $\log(BP_t)$ model ($m = 1$ and 0.025 nominal significance level). Panel B reports the correlation of the break indicator variable for the $\log(RV_t)$ model ($m = 1$ and 0.025 nominal significance level) with the jump component estimate.

Table 2.5: Conditional Mean Forecasts of $\log(RV_{t+1})$, JPY-USD

		AR(5)		AR(10)		Mixed	
		Include	Exclude	Include	Exclude	Include	Exclude
A							
	R^2	0.5888	0.5896	0.5913	0.5919	0.5923	0.5930
	MSE	0.1894	0.1891	0.1883	0.1880	0.1878	0.1875
	MAE	0.3250	0.3245	0.3245	0.3242	0.3234	0.3230
B							
	R^2	0.6048	0.6054	0.6117	0.6123	0.6121	0.6127
	MSE	0.1656	0.1654	0.1624	0.1622	0.1623	0.1620
	MAE	0.3123	0.3118	0.3104	0.3102	0.3095	0.3091
C							
	R^2	0.5888	0.5892	0.5913	0.5914	0.5923	0.5927
	MSE	0.1894	0.1892	0.1883	0.1882	0.1878	0.1876
	MAE	0.3250	0.3244	0.3245	0.3244	0.3234	0.3230
D							
	R^2	0.6807	0.6410	0.6441	0.6440	0.6474	0.6478
	MSE	0.1449	0.1448	0.1437	0.1438	0.1417	0.1416
	MAE	0.2979	0.2974	0.2971	0.2971	0.2947	0.2943

The R^2 is from a regression over the forecast evaluation period of $Y = \beta_0 + \beta_1 X + \epsilon$ where Y is the logarithm of realized volatility and X is the forecast. MSE and MAE denote mean squared error and mean absolute error of the forecast, respectively. Two methods are considered. Method Include is when breaks are not identified and the forecast is based on all pre-forecast data, from observation 1011. Method Exclude is when breaks are identified by the S test with $m = 1$ and the associated observations removed from the sample in which parameter estimates for forecasts are based. The forecast evaluation period starts at observation 2750 and continues until observation 4001. The model estimations are based on data going back to observation 1011 in the dataset. Panels A and C record results for 0.01 and 0.025 nominal significance levels, respectively, computed over all days during the forecast evaluation period. Panels B and D record results for 0.01 and 0.025 nominal significance levels, respectively, computed over all structurally stable days in the forecast evaluation period.

Table 2.6: Conditional Mean Forecasts of RV_{t+1}, JPY-USD

		AR(5)		AR(10)		Mixed	
		Include	Exclude	Include	Exclude	Include	Exclude
A							
	R^2	0.2791	0.2809	0.2809	0.2815	0.2809	0.2815
	MSE	1.015	1.012	1.012	1.011	1.012	1.012
	MAE	0.2922	0.2939	0.2930	0.2950	0.2936	0.2954
B							
	R^2	0.5870	0.5879	0.5915	0.5919	0.5938	0.5942
	MSE	0.1512	0.1509	0.1493	0.1492	0.1485	0.1483
	MAE	0.2258	0.2253	0.2242	0.2240	0.2230	0.2229
C							
	R^2	0.2791	0.2777	0.2809	0.2794	0.2809	0.2790
	MSE	1.015	1.017	1.012	1.014	1.012	1.015
	MAE	0.2922	0.2942	0.2930	0.2959	0.2936	0.2961
D							
	R^2	0.6386	0.6376	0.6383	0.6359	0.6478	0.6456
	MSE	0.1192	0.1196	0.1208	0.1216	0.1152	0.1159
	MAE	0.2082	0.2080	0.2090	0.2091	0.2046	0.2049

The R^2 is from a regression over the forecast evaluation period of $Y = \beta_0 + \beta_1 X + \epsilon$ where Y is realized volatility and X is the respective forecast. MSE and MAE denote mean squared error and mean absolute error of the forecast, respectively. Two methods are considered. Method Include is when breaks are not identified and the forecast is based on all pre-forecast data, from observation 1011. Method Exclude is when breaks are identified by the S test with $m = 1$ and the associated observations removed from the sample in which parameter estimates for forecasts are based. The forecast evaluation period starts at observation 2750 and continues until observation 4001. The model estimations are based on data going back to observation 1011 in the dataset. Panels A and C record results for 0.01 and 0.025 nominal significance levels, respectively, computed over all days during the forecast evaluation period. Panels B and D record results for 0.01 and 0.025 nominal significance levels, respectively, computed over all structurally stable days in the forecast evaluation period.

Table 2.7: Empirical Coverage of JPY-USD One Day Ahead Volatility Forecast Percentiles

		AR(5)		AR(10)		Mixed	
	Percentile	Include	Exclude	Include	Exclude	Include	Exclude
A							
	0.05	0.0312	0.0375	0.0264	0.0343	0.0272	0.0359
	0.10	0.0615	0.0687	0.0623	0.0687	0.0631	0.0703
	0.25	0.2157	0.2181	0.2236	0.2236	0.2204	0.2188
	0.50	0.5495	0.5335	0.5519	0.5367	0.5471	0.5319
	0.75	0.8043	0.7819	0.8035	0.7851	0.8075	0.7819
	0.90	0.9073	0.8978	0.9113	0.8970	0.9089	0.8954
	0.95	0.9489	0.9329	0.9465	0.9345	0.9449	0.9321
B							
	0.05	0.0312	0.0415	0.0264	0.0375	0.0272	0.0391
	0.10	0.0615	0.0743	0.0623	0.0775	0.0631	0.0767
	0.25	0.2157	0.2196	0.2236	0.2244	0.2204	0.2188
	0.50	0.5495	0.5072	0.5519	0.5224	0.5471	0.5120
	0.75	0.8043	0.7612	0.8035	0.7572	0.8075	0.7644
	0.90	0.9073	0.8802	0.9113	0.8842	0.9089	0.8842
	0.95	0.9489	0.9217	0.9465	0.9233	0.9449	0.9217

This table reports results on the empirical coverage of the percentiles for the Gaussian distribution of $\log(RV_{t+1})$ for the JPY-USD market. The empirical coverage is the percentage of these days in which the realized volatility was less than the forecast percentile. Two methods are considered. Method Include is when breaks are not identified and the forecast is based on all pre-forecast data, from observation 1011. Method Exclude is when breaks are identified by the S test with $m = 1$ and the associated observations removed from the sample in which parameter estimates for forecasts are based. The forecast evaluation period starts at observation 2750 and continues until observation 4001. The model estimations are based on data going back to observation 1011 in the dataset. Panels A and B record results for 0.01 and 0.025 nominal significance levels, respectively.

Figure Legends

Figure 2.1: Breaks for AR(5) model, JPY-USD Volatility, sig. level = 0.025

Panel A is the time series of log-realized volatility versus time. Panels B – D display the identified break point observations as a vertical line using a block length of $m = 1, 5$, and 10 respectively, and the AR(5) model of log-realized volatility.

Figure 2.2: Breaks for AR(5) model, DEM-USD Volatility, sig. level = 0.01

Panel A is the time series of log-realized volatility versus time. Panels B – D display the identified break point observations as a vertical line using a block length of $m = 1, 5$, and 10 respectively, and the AR(5) model of log-realized volatility.

Figure 2.3: Breaks for AR(10) model, JPY-USD Bi-Power Variation, sig. level = 0.025

Panel A is the time series of log-bi-power variation versus time. Panels B – D display the identified break point observations as a vertical line using a block length of $m = 1, 5$, and 10 respectively, and the AR(5) model of log-bi-power variation.

Figure 2.4: Frequency of Breaks in last 20 days with m=1

Each panel displays the number of breaks identified with $m = 1$ for the past 20 observations at each point in time. For example, in $[t - 20, t - 1]$ if 4 breaks were found the figure would have a vertical line going to 4 at time $t - 1$. Panels A – D report the frequency of breaks in a window of 20 days for different measures of volatility and time series models.

Figure 2.5: Empirical Coverage for AR(10) model, Absolute Error

This figure plots the probability associated with each percentile minus the percent of observations that lie below the conditional quantile of the one period ahead forecast density.

Figure 2.1: Breaks for AR(5) model, JPY-USD Volatility, sig. level = 0.025

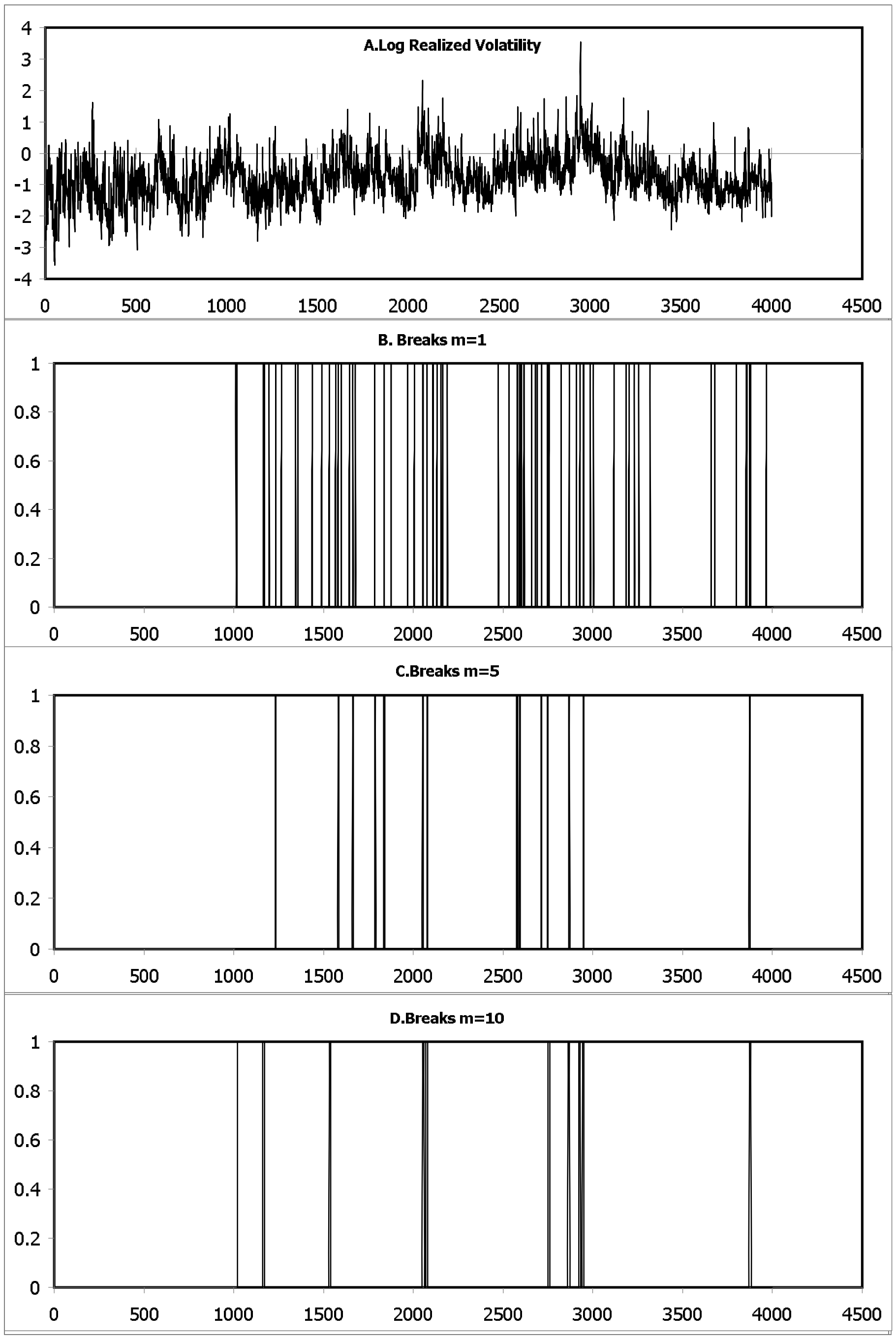


Figure 2.2: Breaks for AR(5) model, DEM-USD Volatility, sig. level = 0.01

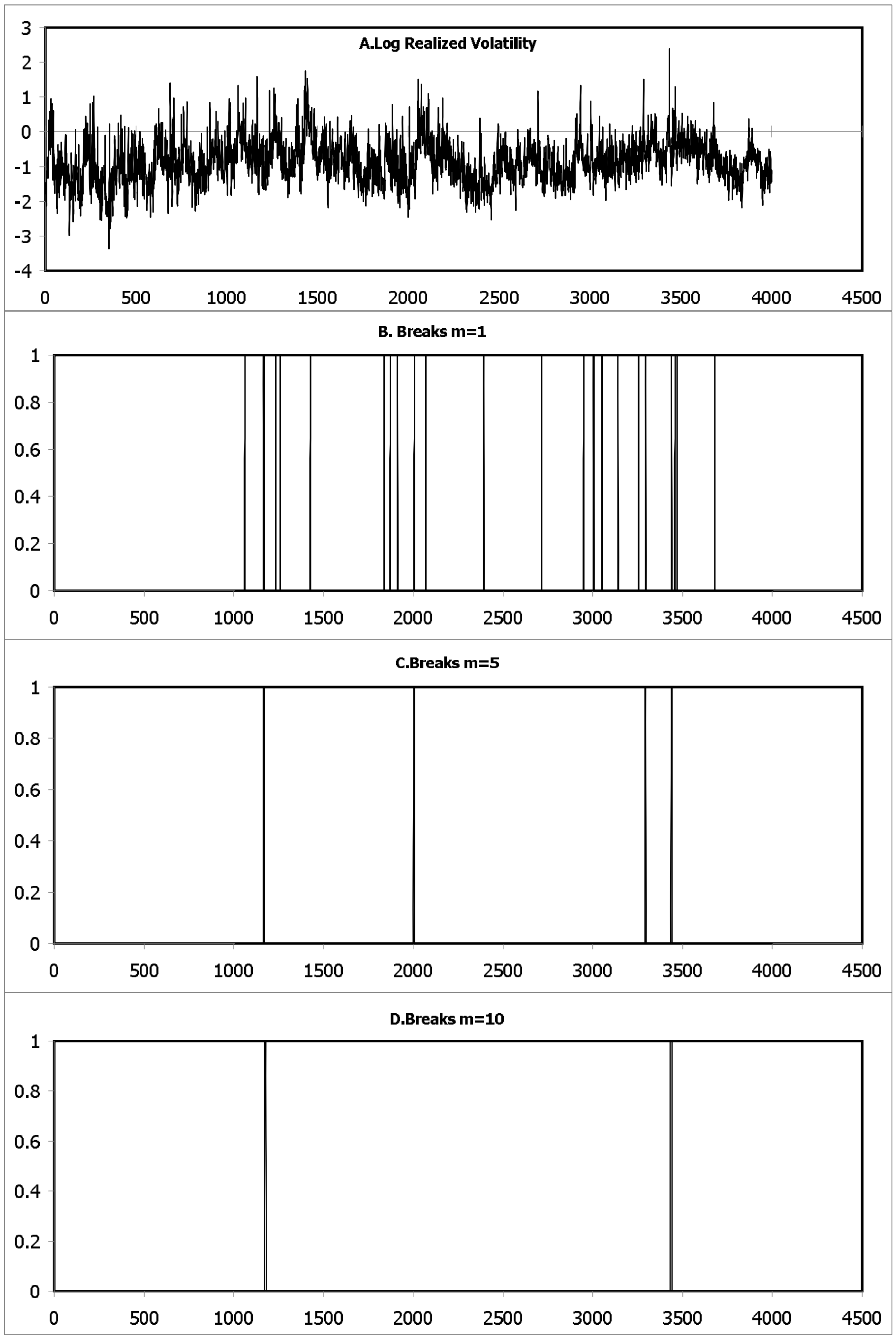


Figure 2.3: Breaks for AR(10) model, JPY-USD Bi-Power Variation, sig. level = 0.025

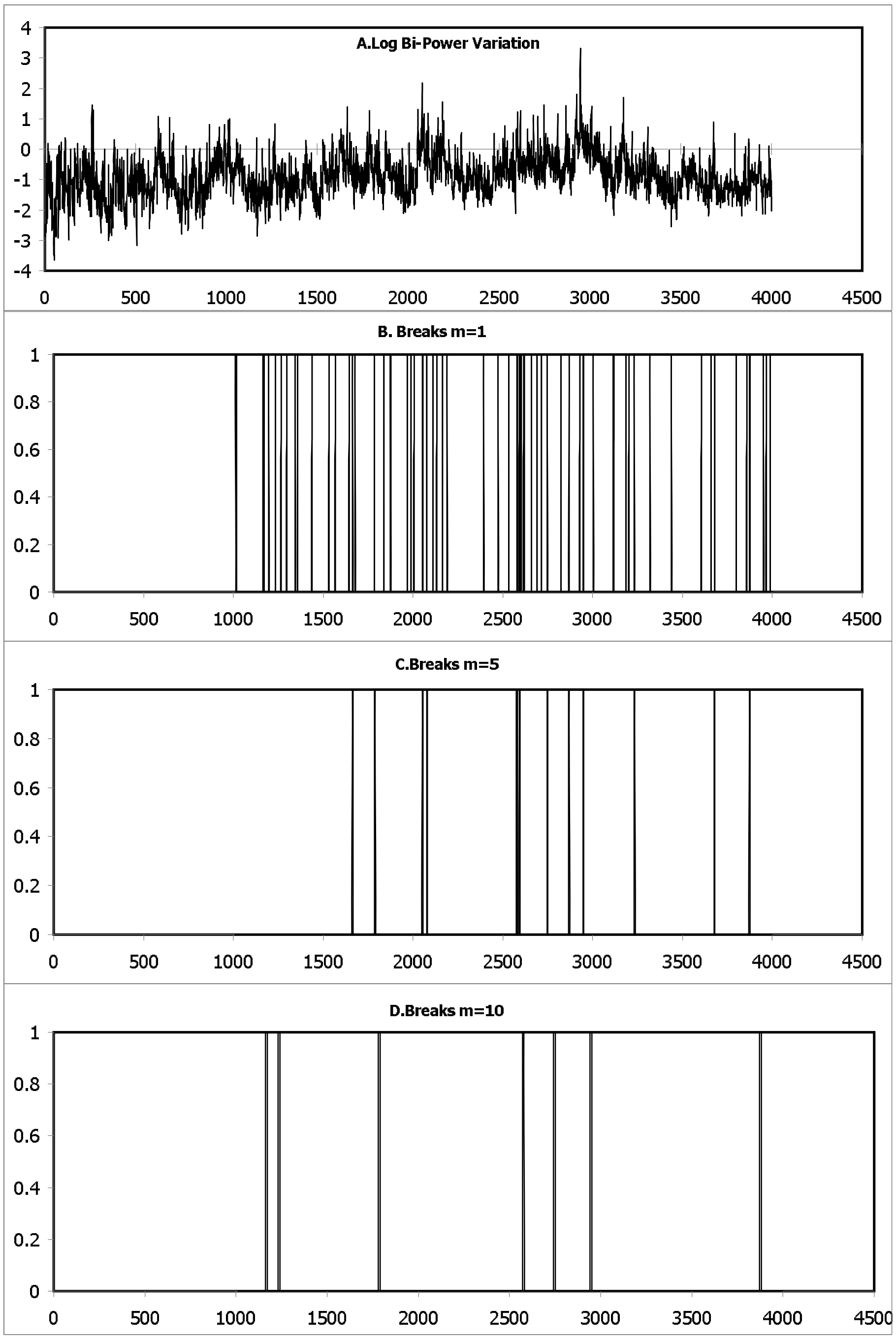


Figure 2.4: Frequency of Breaks in last 20 days with m=1

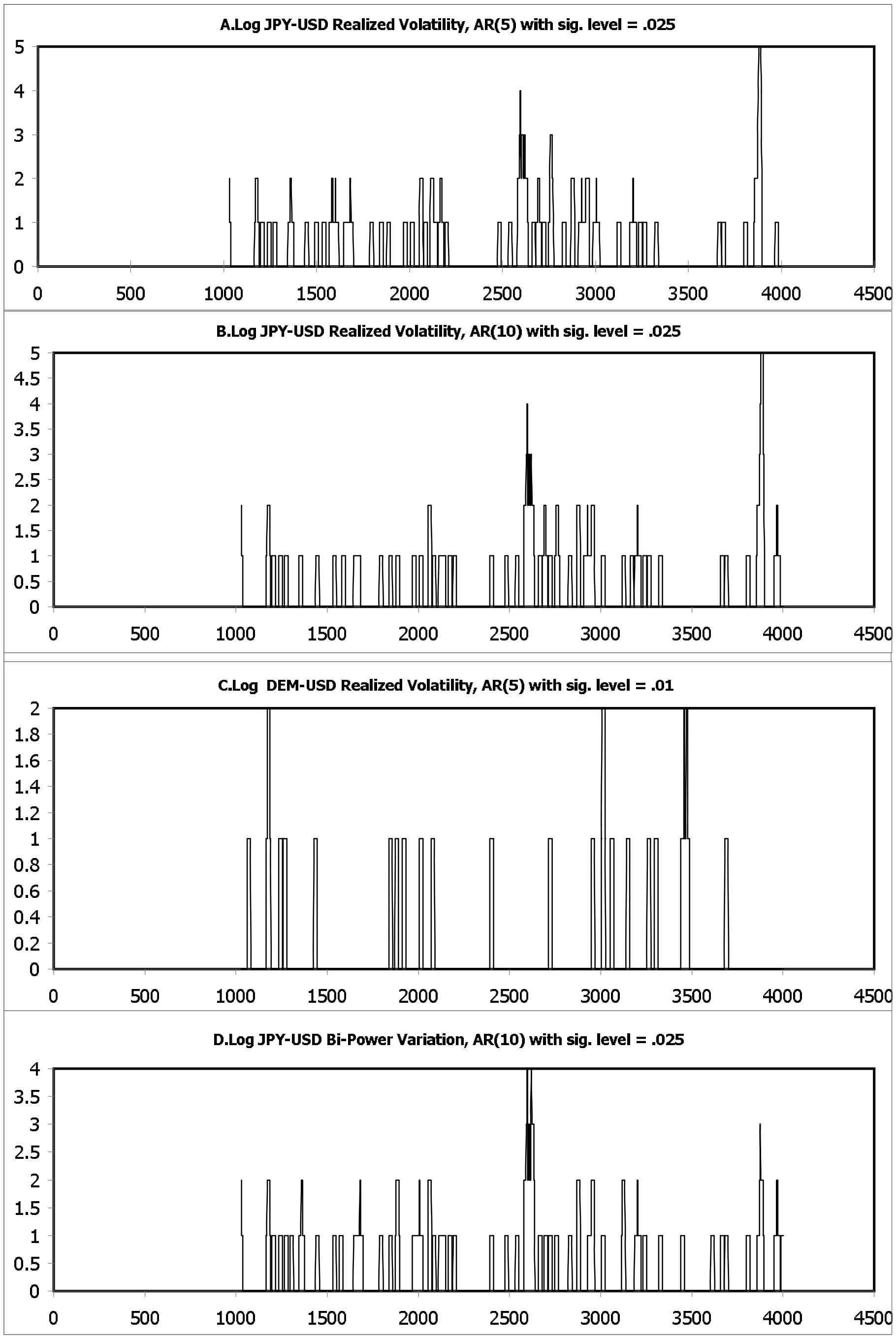
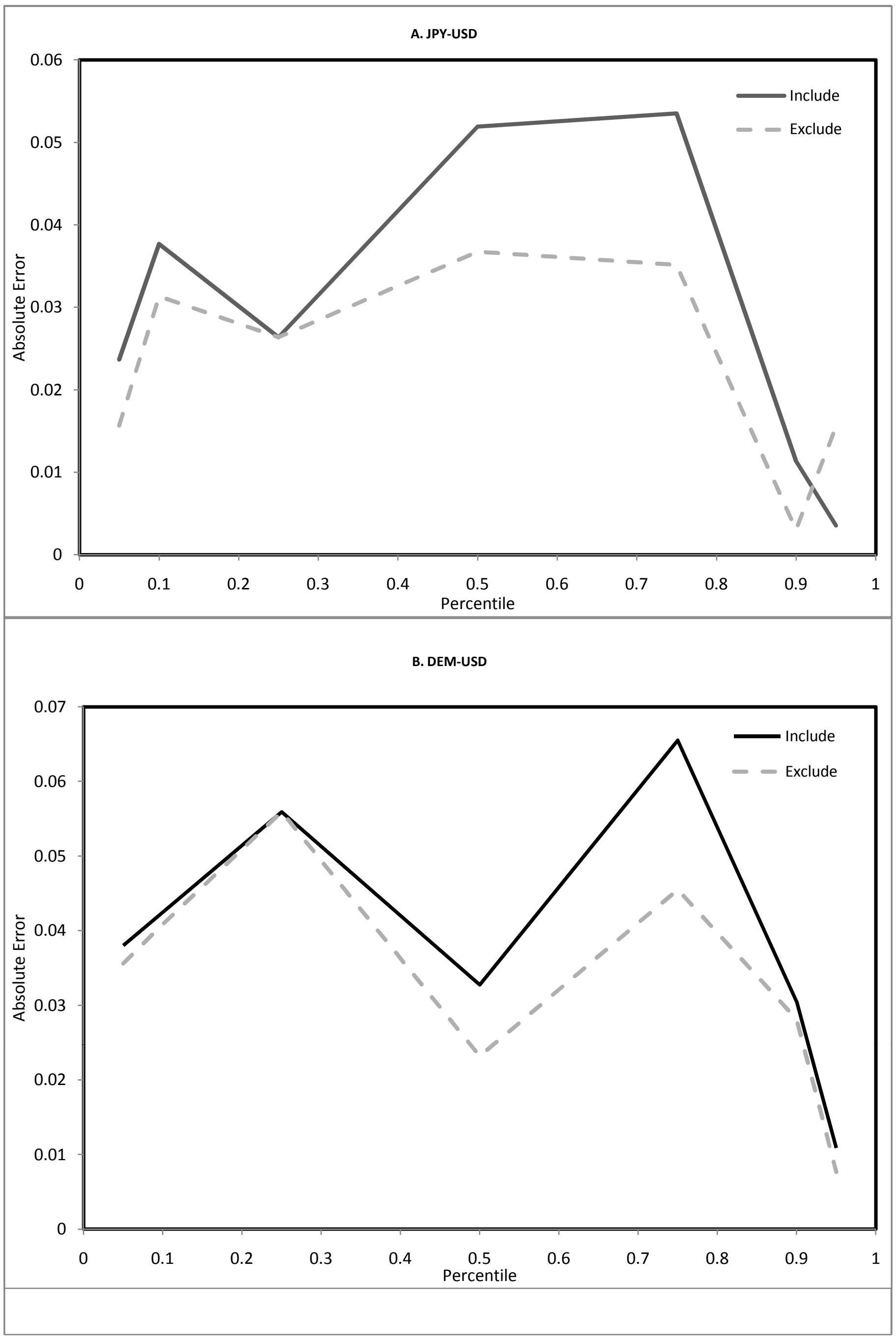


Figure 2.5: Empirical Coverage for AR(10) model, Absolute Error



Chapter 3

Forecasting Stock Return Volatility at the Quarterly Frequency

3.1 Introduction

There is a vastly rich literature documented on volatility forecasting at the horizon from one day to one month, while the academic research on the long term stock price volaitlity forecasting is relatively silent. Christoffersen and Diebold (2000) and West and Cho (1995) found long term volatility is hard to forecast. However,forecasting return volatility at horizons such as the quarterly frequency play an important role in asset pricing and financial risk management. Practitioners always are interested in volatility forecast to construct volatility curve with the maturities from one week to one year, especially one week, one month and one quarter are the most crucial horizons. Threfore, an accurrate quarterly forecast of volatility is economically important for decision makers. Mayhew (1995) found quarterly forecasts of return volatility implied from option prices are heavily used by market participants, though these implied volatilities are based on market prices which may be subject to mis-pricing and are not always readily available. Often volatility forecasts based on historical time series of returns are also utilized, based on autoregressive specifications following Engle (1982) and Bollerslev (1986). Recent improvements in volatility forecasting have occurred by utilizing the realized volatility measurement and modeling approaches of Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2002) and Andersen et al. (2003) The realized volatility literature has primarily focused on short horizon volatility forecasting ranging from daily to monthly frequencies as there is a high degree of predictability at these frequencies, see for example, Koopman et al. (2005), Andersen et al. (2007), Corsi (2009) and Martens et al. (2009). Ghysels et al. (2009) explore longer range return volatility forecasting, demonstrating predictability at the quarterly horizon, and showing that the mixed-data sampling (MIDAS) approach, introduced by Ghysels et al. (2005 and 2006), has superior forecasting performance relative to commonly utilized models such as GARCH.

In this chapter we study stock return volatility forecasting at the quarterly frequency. We demonstrate superior forecasting performance from a simple autoregressive model with one lag of quarterly realized volatility AR(1), that dominates the MIDAS approach. The quarterly realized volatility for the AR(1) model estimation is computed from daily returns and thus this quarterly forecasting procedure can be applied to a wide class of assets. The assets directly analyzed in this study are stocks from the Dow Jones Industrial Average Index (DJIA) due to the access of reliable thirty minute returns for quarterly realized volatility measurement for the purposes of measuring forecast accuracy. Over our sample of stocks we find that the simple autoregressive model with one lag of quarterly realized volatility has a lower mean-squared-forecast-error and mean-absolute-forecast-error than the MIDAS forecasts. Since the MIDAS models are more complicated to estimate, than the simple autoregressive models, often involving non-linear estimation methods, we conclude that MIDAS are an inferior forecasting method for quarterly volatility, without the need to show statistically significant differences in the forecasts of these two approaches.

Parsimony is one of the key considerations when constructing a volatility model. The advantage of parsimonious volatility models has been discussed in Hansen and Lunde (2005), where it is found none of the 330 complicated models can outperform GARCH(1,1). This demonstrates the crucial fact that the simple model could perform better or equally well and the complicated models fail to provide additional benefit. The results of this chapter reinforce this important principle in forecasting in that relatively parsimonious models often deliver the superior forecasting performance. Andersen et al. (2003) demonstrate this principle for short horizon volatility forecasts finding the dominant model to be a simple autoregressive model of daily realized volatility. This dominance was demonstrated over an extensive range of commonly used time series forecasting models for volatility, including non-linear models. However, Ghysels et al. (2006) with MIDAS models and Corsi (2009) with a Heterogeneous Autoregressive (HAR) model do find some improvements in short horizon volatility predictions, relative to simple autoregressive specifications of daily realized volatility.

This chapter is organized as follows. The next section reviews realized volatility measurement and Section 3.3 describes our dataset of DJIA stocks. Section 3.4 discusses the forecasting approaches and Section 3.5 details the empirical results of our study. Section 3.6 contains concluding remarks.

3.2 Volatility Measurement

Please refer to Section 2.3 for the detailed literature review of volatility measurement. As shown in Equation 2.14 , when the return sampling frequency tends to infinity, the realized volatility estimator approaches the quadratic variation of the return, see Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004b). This result of RV_t being a consistent estimator of QV_t as $N \to \infty$, is the theoretical motivation behind realized volatility measurement, however, there are important considerations in regard to the return sampling frequency when realized volatility is computed in practice. Firstly, market microstructure noise (e.g. discreteness of prices and bid/ask bounce) can result in inaccurate high frequency return measurement, thus a balance needs to be reached between a sufficiently high N and a reliable $r_{t,i}$. Bollerslev et al. (2007) suggests that a sampling frequency of 22.5 minutes mitigates the effect of "noise" for all of their 40 stocks. Thus, taking a cautious approach with our Dow stocks, we choose a sampling frequency of 30 minutes for our intraday returns. Unlike foreign exchange market, stock exchange does not open 24 hours a day. Martens et al. (2008) has documented that overnight volatility reprensents an important part of total daily volatility. Therefore, it's necessary to incorporate overngiht stock returns for accurately measuring volatility. For the purposes of ex-post volatility measurement in forecast evaluation, this chapter computes realized volatility over a calendar quarter using 30 minute intraday returns and the overnight return. i.e. for each trading day in the quarter, we compute the 30 minute intraday returns and overnight return. These returns are then squared and summed over the quarter.

3.3 Data

Our dataset consists of stocks from the Dow Jones Industrial Average Index (DJIA). Daily data from January 1, 1975 to July 31, 2008, consisting of stock returns, open and close prices, are obtained from the Center for Research in Security Prices (CRSP), with adjustments made for corporate actions, such as dividends, splits etc. High-frequency data from August 1, 1997 to July 31, 2008, consisting of 30-minute intraday price data, sampled from 9:30AM to 3:30PM, are obtained from price-data[1]. The following stocks, Home Depot, Citigroup, Microsoft, AT&T Inc., Chevron Corp., Verizon Communication and Exxon Mobil Corp., are excluded due to incomplete return time series over the study period, leaving 23 stocks for our analysis.

3.4 Forecasting Approaches

We focus our study on three forecasting approaches for quarterly volatility. These are forecasts from constant volatility models, autoregressive realized volatility models, and MIDAS models. Constant volatility models are chosen as an initial benchmark and also because these are often used by practitioners, see for example, common estimates of Value-at-Risk (VaR). Autoregressive realized volatility models are chosen based on their recent popularity in forecasting short range volatility, see Andersen et al. (2003). MIDAS models are chosen as Ghysels et al. (2009) recently demonstrate these models dominating other commonly used forecasting approaches for quarterly volatility such as GARCH. We now briefly discuss each of these three approaches.

[1] www.grainmarketresearch.com

3.4.1 Constant Volatility Models

Constant volatility models forecast volatility from an average volatility measurement over a prior time period. Our constant volatility quarterly forecasts are computed as the average quarterly realized volatility computed over the prior l quarters, where the quarterly realized volatility is computed from daily returns. With the i^{th} stocks quarterly volatility forecasting equation being:

$$\sigma^2_{i,t+1} = \frac{1}{l}\sum_{k=0}^{l-1}\sigma^2_{i,t-k} \tag{3.1}$$

3.4.2 Autoregressive Realized Volatility Models

The most commonly estimated model in the realized volatility literature is the the autoregressive model of logged realized volatility with p lags defined as:

$$ln(\sigma^2_{i,t+1}) = \phi_{i,0} + \sum_{k=1}^{p}\phi_{i,k}ln(\sigma^2_{i,t+1-k}) + \epsilon_{i,t+1} \tag{3.2}$$

In our model estimations, $\sigma^2_{i,t}$ is the quarterly realized volatility computed from daily returns for stock i during quarter t.

3.4.3 MIDAS Models

The mixed-data sampling (MIDAS) approach, introduced by Ghysels et al. (2005 and 2006) has recently been advocated as an approach to quarterly volatility forecasting that dominates other commonly used approaches, see Ghysels et al. (2009). In the MIDAS approach to quarterly volatility forecasting, the forecasting regression can be formulated as follows:

$$\sigma^2_{i,t+1} = \mu_i + \phi_i \sum_{j=0}^{j^{max}} b_i(j,\theta) r^2_{t-j} + \epsilon_{i,t+1} \tag{3.3}$$

where $\sigma^2_{i,t+1}$ is a measure of quarterly volatility (in our model estimations quarterly realized volatility is computed by the sum of squared daily returns within the quarter) and the regressors, $r^2_{t-j}, j = 0, \ldots, j^{max}$ are measured at a higher frequency, (in our our applications we use daily squared returns.) The weighting function, $b_i(j,\theta)$, is parameterized by a low-dimensional parameter vector θ. The intercept μ_i, slope ϕ_i and weighting parameters θ are typically estimated with a Gaussian likelihood as quasi-maximum likelihood estimation, which we follow in our model estimations.

The following weighting functions have been suggested by Ghysels et al. (2005 and 2006) and Ghysels et al. (2009) and are empirically evaluated in Section 5.

1. Exponential:

$$b_i(j,\theta_1,\theta_2) = \frac{exp\{\theta_1 j + \theta_2 j^2\}}{\sum_{k=0}^{\infty} exp\{\theta_1 k + \theta_2 k^2\}} \tag{3.4}$$

   which can produce a variety of different decay patterns.

2. Beta:

$$b_i(j,\theta_1,\theta_2) = \frac{f(\frac{j}{j^{max}},\theta_1;\theta_2)}{\sum_{k=1}^{j^{max}} f(\frac{k}{j^{max}},\theta_1;\theta_2)} \tag{3.5}$$

where:

$$f(x, a, b) = \frac{x^{a-1}(1-x)^{b-1}\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \tag{3.6}$$

$$\Gamma(a) = \int_0^\infty e^{-x}x^{a-1}dx \tag{3.7}$$

which can also produce a variety of different decay patterns. Equation 3.6 defines the Gamma function.

3. Hyperbolic:

$$b_i(j, \theta) = \frac{g(\frac{j}{j^{max}}, \theta)}{\sum_{k=1}^{j^{max}} g(\frac{k}{j^{max}}, \theta)} \tag{3.8}$$

where $g(j, \theta) = \Gamma(j + \theta)/(\Gamma(j + 1)\Gamma(\theta))$ which can be written equivalently as $g(0, \theta) = 1$ and $g(j, \theta) = (j + \theta - 1)g(j - 1, \theta)/j$, for $j \geq 1$. This specification is not as flexible as the beta weighting function.

In addition to the above three functional forms, two restricted versions of the exponential and beta specifications, have also been suggested. "Exp Rest" is when the constraint of $\theta_2 = 0$ is imposed on the exponential specification and "Beta Rest" is when the constraint of $\theta_1 = 1$ is imposed on the beta specification. These restricted specifications lead to a slowly decaying pattern of the weighting functions. For further details, see Ghysels et al. (2009).

3.5 Empirical Results

The descriptive statistics is displayed in Table 3.1 for the stock daily returns. The mean of returns across all stocks are no different to zero, however, the standard deviation varies.There is excessive kurtosis for the return series. Figure 3.1 displays the quarterly realized volatility for our 23 DJIA stocks, computed both from 30-minute and daily returns. There is a close correspondence between these two volatility measures, though the volatility measure from daily returns displays some measurement error with an upward bias. Figures 3.6 and 3.6 display the Autocorrelation Functions (ACFs) and Partial Autocorrelation Functions (PACFs) of quarterly logarithmic realized volatility, computed from daily returns, for each stock over the sample period from Q4, 1997 to Q2, 2008. The quarterly logarithmic realized volatility have gradual declining autocorrelations for all stocks in our sample and most of the PACFs cut off at lag 1.

We next empirically evaluate the three approaches to quarterly volatility forecasting, discussed in the previous section. One quarter ahead forecasts from these models over the forecast evaluation period from Q4, 1997 to Q2, 2008 are measured against the quarterly realized volatility computed from 30-minute returns and overnight returns. The prediction accuracy is evaluated on the basis of mean squared error (MSE) and mean absolute error (MAE) for each stock. The MSE and MAE are computed as follows:

$$MSE = \frac{1}{m}\sum_{r=1}^{m}(\widetilde{\sigma^2_{i,r}} - \widehat{\sigma^2_{i,r}})^2 \tag{3.9}$$

$$MAE = \frac{1}{m}\sum_{r=1}^{m}|\widetilde{\sigma^2_{i,r}} - \widehat{\sigma^2_{i,r}}| \tag{3.10}$$

where $\widetilde{\sigma^2_{i,r}}$ represents the one-quarter-ahead volatility forecast made at the end of quarter $r-1$ for stock i and $\widehat{\sigma^2_{i,r}}$ denotes the realized volatility computed from 30-minute returns and overnight returns in quarter r for stock i.

The constant model forecasts are from in-sample estimation periods of 1, 2, 3, 4, 5 and 6 quarters. While the AR model forecasts are from in-sample estimation periods of 20, 30, 40, 50, 60, 70 and 80 quarters. Finally, the MIDAS model forecasts are from an in-sample estimation period of 68 quarters, with MIDAS lag lengths of 40, 60, 80, 100, 150 and 200 trading days.

Table 3.2 and 3.3 display the performance of the one-quarter-ahead volatility forecasts. The values of MSE and MAE are the average values over all 23 stocks for each model. The AR(1) model with an in-sample estimation period of 70 quarters produces the lowest MSE and MAE, 2.5123 and 0.9903, respectively. Not surprisingly given the well known time-variation in volatility, the constant model forecasts perform poorly. Lastly, the MIDAS models demonstrate varying degrees of performance. The unrestricted beta, restricted beta and unrestricted exponential model forecast errors are worse than some of the constant models. Whereas the restricted exponential and hyperbolic model forecast errors are less than the constant models. Relative to the other MIDAS specifications, the hyperbolic model with a lag length of 100 trading days produces the lowest MSE and MAE, 2.7146 and 1.0802, respectively.

Given the lower MSE and MAE results for the AR(1) model, its parsimonious specification and ease of estimation, it dominants the other models for quarterly volatility forecasting. Table 3.4 and 3.5 display the MSE and MAE for the AR(1) one-quarter-ahead volatility forecast for each stock, with in-sample estimation periods ranging from 20 to 80 quarters. For almost all the stocks, the lowest forecast error is from in-sample estimation periods of between 60 and 80 quarters. This suggests that investors with only daily returns can employ the simple AR(1) model with 15 to 20 years historical data to forecast quaterly volatility ahead.

3.6 Conclusions

There is a long tradition in the forecasting literature of utilizing parsimonious time series models. Often these models produce the most accurate forecasts. This forecasting principle in the volatility literature was demonstrated by Andersen et al. (2003) where they found standard autoregressive models of daily realized volatility as the dominant forecasting approach for short range volatility, relative to other more complicated commonly used approaches at that time. However, further research by Ghysels et al. (2006) and Corsi (2009) with MIDAS models did find some forecasting improvements over standard autoregressive models of daily realized volatility. In this chapter we find that for longer range volatility forecasts at the quarterly frequency, an autoregressive model with one lag of quarterly realized volatility is the dominant forecasting model. MIDAS models are considered, though they are found to generate inferior forecasts at the quarterly horizon.

Table 3.1: Descriptive Statistics for stocks and Index

Company Name	Mean	Stdev	Min	Max	Skew	Kurtosis
AMERICAN EXPRESS	0.001	0.021	-0.262	0.186	-0.110	10.780
AMER INTL GRP	0.000	0.023	-0.608	0.431	-2.354	122.740
INTEL CORP	0.001	0.027	-0.220	0.264	0.024	7.554
BANK OF AMERICA CORP	0.001	0.021	-0.262	0.272	0.269	21.599
HOME DEPOT INC	0.001	0.024	-0.287	0.226	-0.270	13.587
MICROSOFT CORP	0.001	0.024	-0.301	0.196	-0.132	12.923
ALCOA INC	0.001	0.021	-0.241	0.232	0.203	13.602
BOEING CO	0.001	0.020	-0.176	0.155	0.089	7.711
CATERPILLAR INC	0.001	0.019	-0.216	0.147	-0.139	9.946
JP MORGAN CHASE	0.001	0.022	-0.277	0.214	0.149	14.251
COCA-COLA CO	0.001	0.016	-0.247	0.197	0.039	16.464
CITIGROUP INC	0.001	0.026	-0.264	0.578	1.803	58.267
DISNEY (WALT) CO	0.001	0.020	-0.291	0.191	-0.267	15.871
DU PONT	0.000	0.017	-0.183	0.115	-0.031	8.237
GENERAL ELECTRIC	0.001	0.016	-0.175	0.136	0.010	10.521
GENERAL MOTORS	0.000	0.022	-0.311	0.351	0.458	30.822
HEWLETT-PACKARD	0.001	0.023	-0.203	0.209	0.135	9.149
IBM	0.000	0.017	-0.230	0.132	0.040	13.518
JOHNSON&JOHNSON	0.001	0.015	-0.184	0.122	-0.119	9.780
MCDONALDS CORP	0.001	0.017	-0.166	0.109	0.083	7.570
MERCK & CO	0.001	0.017	-0.268	0.130	-0.503	15.504
3M CO	0.001	0.015	-0.260	0.115	-0.444	17.897
PFIZER	0.001	0.018	-0.173	0.102	-0.053	6.588
PROCTER & GAMBLE CO	0.001	0.015	-0.314	0.222	-1.379	48.259
A T & T INC	0.001	0.017	-0.219	0.202	0.117	14.766
UNITED TECH CORP	0.001	0.017	-0.282	0.136	-0.400	14.960
WAL-MART STORES	0.001	0.020	-0.118	0.124	0.330	6.094
DJIA Index	0.000	0.011	-0.256	0.105	-1.538	44.574

This table presents the descriptive statistics for daily returns of all 23 stocks and Dow Jones Industrial Average Index. We show the mean, standard deviation, minimum, maximum, skew and kurtosis. The data is obtained from the Center for Research in Security Prices (CRSP), with adjustments made for corporate actions, such as dividends, splits etc. Daily return series starts from January 1, 1975 and ends on July 31, 2008.

Table 3.2: Average MSE of One-Quarter-Ahead Volatility Forecasts

A

	In-Sample Size					
	1Q	2Q	3Q	4Q	5Q	6Q
Constant Model	3.8085	3.4697	3.3156	3.2622	3.2757	3.1914

B

	In-Sample Size						
	20Q	30Q	40Q	50Q	60Q	70Q	80Q
AR(1)	3.0848	2.9676	2.7900	2.5969	2.5262	**2.5123**	2.5263
AR(2)	3.6076	3.1639	2.9240	2.7473	2.6097	2.5694	2.5420
AR(3)	4.1923	3.3301	2.9829	2.8288	2.7289	2.6849	2.6263
AR(4)	5.7534	3.6887	3.1576	2.8935	2.7470	2.7176	2.6599
AR(5)	9.6888	4.1559	3.3163	2.9525	2.7800	2.7838	2.7023

C

	MIDAS Specification				
MIDAS Lag Length	BETA	BETA REST	EXP	EXP REST	HYPERB
40	3.7946	3.2419	3.3573	3.3314	2.8993
60	3.5392	3.2159	3.4558	3.2205	2.7218
80	3.5257	3.2190	3.5150	3.5670	3.0740
100	3.5352	3.1776	3.4518	3.2003	2.7146
150	3.5675	3.1660	3.3944	3.2325	2.7134
200	3.5165	3.0004	3.4479	3.2185	2.7332

For the AR models, the in-sample estimation period is expressed in number of quarters. The constant model forecast is the average of realized volatility over the past quarters. The MIDAS specifications include the unrestricted beta model denoted by BETA, the restricted beta model, denoted by BETA REST, the unrestricted exponential model, denoted by EXP, the restricted exponential model, denoted by EXP REST and the hyperbolic model, denoted by HYPERB. The MIDAS estimation is based on data over the past 68 quarters. The forecast evaluation period is from 1997 Q4 to 2008 Q2. Values are computed by averaging over stocks and the minimum value is highlighted.

Table 3.3: Average MAE of One-Quarter-Ahead Volatility Forecasts

A

	In-Sample Size					
	1Q	2Q	3Q	4Q	5Q	6Q
Constant Model	1.1790	1.1583	1.1495	1.1560	1.1760	1.1758

B

	In-Sample Size						
	20Q	30Q	40Q	50Q	60Q	70Q	80Q
AR(1)	1.1470	1.1202	1.0673	1.0088	**0.9844**	0.9903	0.9920
AR(2)	1.2279	1.1707	1.0963	1.0450	1.0142	1.0107	1.0028
AR(3)	1.2962	1.2008	1.1092	1.0642	1.0326	1.0290	1.0175
AR(4)	1.4036	1.2683	1.1468	1.0757	1.0324	1.0294	1.0211
AR(5)	1.6438	1.3567	1.1982	1.1096	1.0516	1.0563	1.0418

C

	MIDAS Specification				
MIDAS Lag Length	BETA	BETA REST	EXP	EXP REST	HYPERB
40	1.2186	1.1502	1.2145	1.1653	1.1023
60	1.1755	1.1444	1.2512	1.1648	1.0691
80	1.1780	1.1481	1.2446	1.1805	1.0999
100	1.1850	1.1410	1.2339	1.1632	1.0802
150	1.1715	1.1395	1.2394	1.1660	1.0840
200	1.1584	1.1262	1.2445	1.1778	1.0817

For the AR models, the in-sample estimation period is expressed in number of quarters. The constant model forecast is the average of realized volatility over the past quarters. The MIDAS specifications include the unrestricted beta model denoted by BETA, the restricted beta model, denoted by BETA REST, the unrestricted exponential model, denoted by EXP, the restricted exponential model, denoted by EXP REST and the hyperbolic model, denoted by HYPERB. The MIDAS estimation is based on data over the past 68 quarters. The forecast evaluation period is from 1997 Q4 to 2008 Q2. Values are computed by averaging over stocks and the minimum value is highlighted.

Table 3.4: MSE of one-quarter-ahead AR(1) volatility forecasts

Company	In-Sample Size						
	20	30	40	50	60	70	80
ALCOA INC	2.9559	2.5147	2.0081	1.7017	1.5224	1.5071	**1.4297**
AMER INTL GRP	2.0587	1.8562	1.667	1.381	1.2694	1.2026	**1.1657**
AMERICAN EXPRESS	3.1789	2.5205	2.47	2.2761	2.1428	2.1307	**2.1128**
BOEING CO	2.7732	2.639	2.3018	2.0735	2.025	1.9207	**1.9089**
BANK OF AMERICA CORP	3.6612	3.1982	2.8009	2.7186	2.6712	2.6891	**2.6649**
CATERPILLAR INC	1.6342	1.6259	1.5686	1.4212	1.4076	1.3761	**1.3747**
DU PONT	1.258	1.1595	1.1254	1.099	1.0452	1.0039	**1.0031**
DISNEY (WALT) CO	4.2478	4.0283	3.6786	3.3942	3.3399	**3.2502**	3.3251
GENERAL ELECTRIC	1.3313	1.395	1.4268	1.2081	1.1261	1.1289	**1.1227**
GENERAL MOTORS	2.8533	2.6792	2.5548	2.461	2.4065	2.3852	**2.2528**
HEWLETT-PACKARD	5.7152	5.8366	5.7045	5.3126	**5.2884**	5.3263	5.5681
IBM	1.9014	1.7567	1.7917	1.8391	1.7388	1.6931	**1.661**
INTEL CORP	6.8833	6.2645	5.6497	**5.6457**	5.8801	5.9941	6.2241
JOHNSON & JOHNSON	1.2057	1.1956	1.1618	1.1278	**1.12**	1.1311	1.1376
JP MORGAN CHASE	5.3491	5.1831	5.3796	5.0412	4.6055	**4.5255**	4.5262
COCA-COLA CO	0.6945	0.7316	0.6145	0.5708	**0.49**	0.5114	0.5145
MCDONALDS CORP	1.0199	1.0371	0.9711	0.9388	**0.9196**	0.9227	0.9484
3M CO	0.7533	0.78	0.7409	0.6219	**0.6103**	0.6221	0.6178
MERCK & CO	3.0365	2.9111	2.8639	2.7208	2.7186	**2.703**	2.7776
PFIZER	1.7945	1.5958	1.5046	1.4237	1.3492	1.3289	**1.3084**
PROCTER & GAMBLE CO	12.02	12.4463	11.7686	10.6715	**10.6074**	10.6884	10.7097
UNITED TECH CORP	2.8316	3.3286	2.9907	2.7475	2.5163	**2.4267**	2.433
WAL-MART STORES	1.7938	1.5706	1.4263	1.3329	**1.3034**	1.3152	1.3177

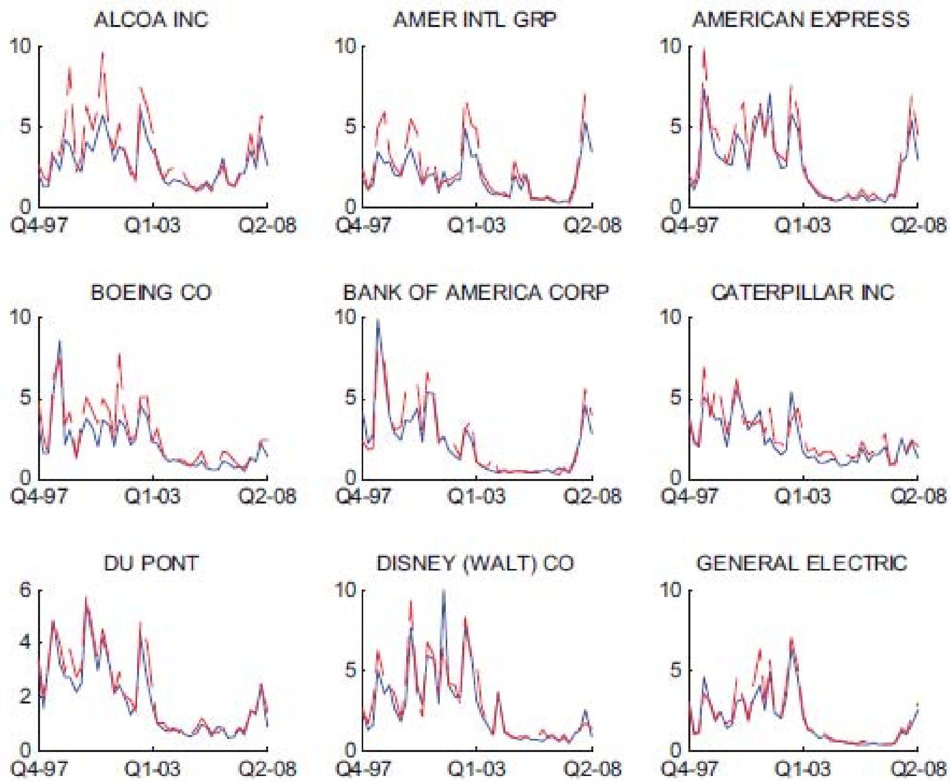
The in-sample estimation period is expressed in number of quarters. The forecast evaluation period is from 1997 Q4 to 2008 Q2 and the minimum value is highlighted.

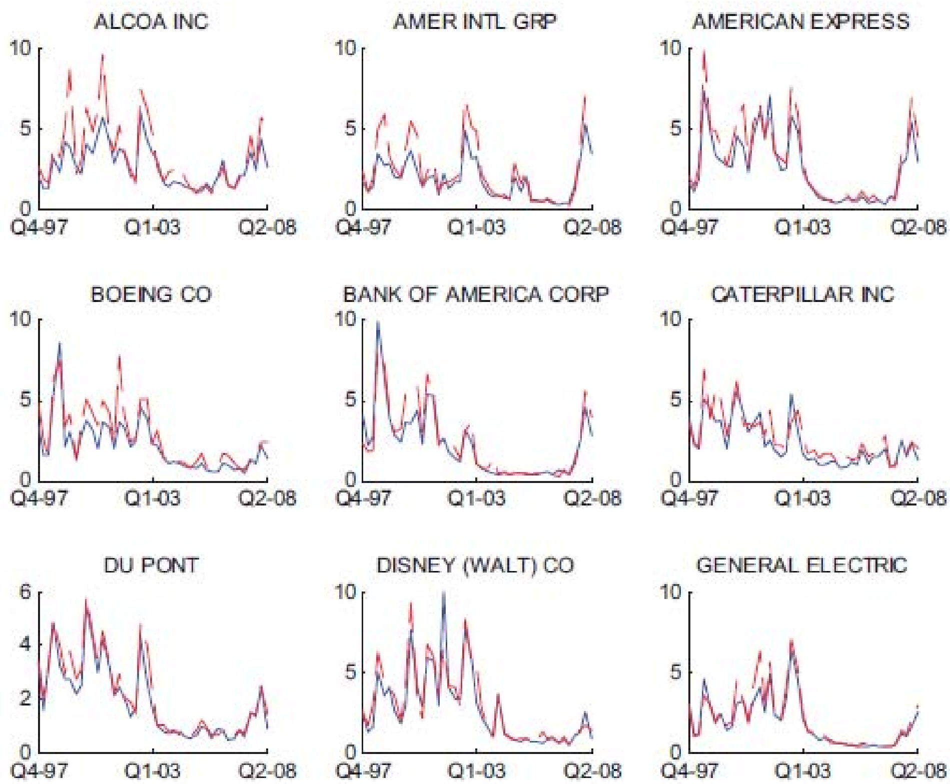
Table 3.5: MAE of one-quarter-ahead AR(1) volatility forecasts

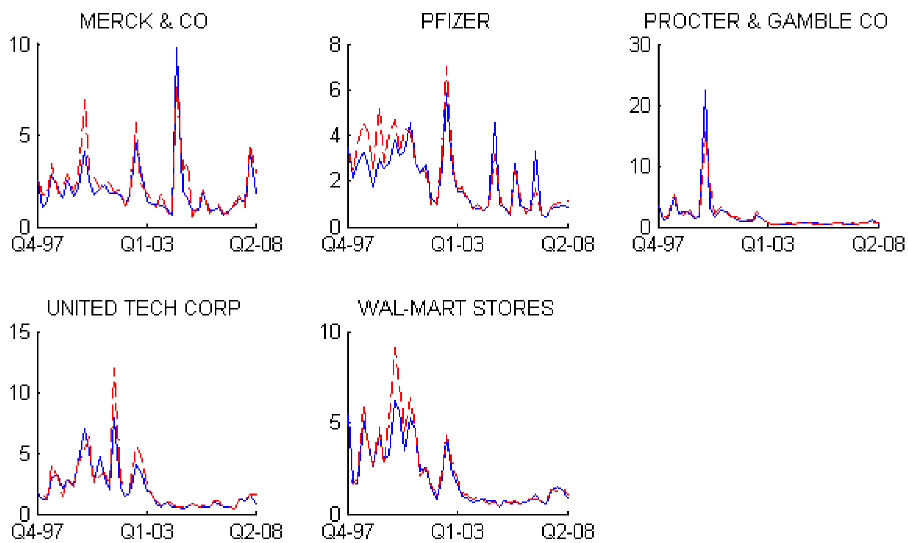
Company	In-Sample Size						
	20	30	40	50	60	70	80
ALCOA INC	1.3575	1.2710	1.1588	1.0700	1.0238	1.0142	**0.9888**
AMER INTL GRP	1.126	1.0593	0.9837	0.9008	0.8639	0.8521	**0.835**
AMERICAN EXPRESS	1.3201	1.1746	1.1461	1.0848	**1.0397**	1.0708	1.0627
BOEING CO	1.2721	1.3288	1.2219	1.1303	1.0993	1.0722	**1.0706**
BANK OF AMERICA CORP	1.2812	1.1316	1.0464	1.0027	**1.0012**	1.0211	1.0085
CATERPILLAR INC	1.0525	1.0568	1.0144	0.9533	0.9451	0.9444	**0.936**
DU PONT	0.8913	0.8215	0.7928	0.7445	**0.7152**	0.7154	0.7221
DISNEY (WALT) CO	1.5079	1.4961	1.3627	1.2453	1.2105	**1.1943**	1.2272
GENERAL ELECTRIC	0.8204	0.8616	0.8265	0.7566	**0.7221**	0.7425	0.7379
GENERAL MOTORS	1.3997	1.3305	1.2915	1.278	1.2522	1.2717	**1.2274**
HEWLETT-PACKARD	1.7187	1.8001	1.7098	1.5764	1.5524	**1.5422**	1.5948
IBM	1.0019	0.9841	0.9894	0.9993	0.9438	0.9399	**0.9272**
INTEL CORP	1.7826	1.7412	**1.6665**	1.6678	1.7455	1.8215	1.8877
JOHNSON&JOHNSON	0.6522	0.6493	0.6188	**0.6054**	0.6112	0.6194	0.6305
JP MORGAN CHASE	1.5077	1.4621	1.5374	1.4756	1.3924	1.3733	**1.3606**
COCA-COLA CO	0.5782	0.561	0.518	0.5111	**0.4776**	0.4984	0.5059
MCDONALDS CORP	0.7699	0.7617	0.7173	0.7055	**0.6905**	0.6982	0.7053
3M CO	0.6968	0.7146	0.6899	0.6088	0.6039	0.6064	**0.6003**
MERCK & CO	1.1546	1.1273	1.0788	0.99	0.968	0.9569	**0.9457**
PFIZER	1.0601	1.0006	0.9571	0.9249	**0.9035**	0.9074	0.9114
PROCTER & GAMBLE CO	1.3431	1.3114	1.2228	1.0425	**1.0163**	1.0499	1.06
UNITED TECH CORP	1.1878	1.2526	1.1752	1.108	1.0583	1.0431	**1.0416**
WAL-MART STORES	0.8998	0.8679	0.821	0.8215	**0.8041**	0.8217	0.8296

The in-sample estimation period is expressed in number of quarters. The forecast evaluation period is from 1997 Q4 to 2008 Q2 and the minimum value is highlighted.

Figure 3.1: Quarterly Realized Volatility

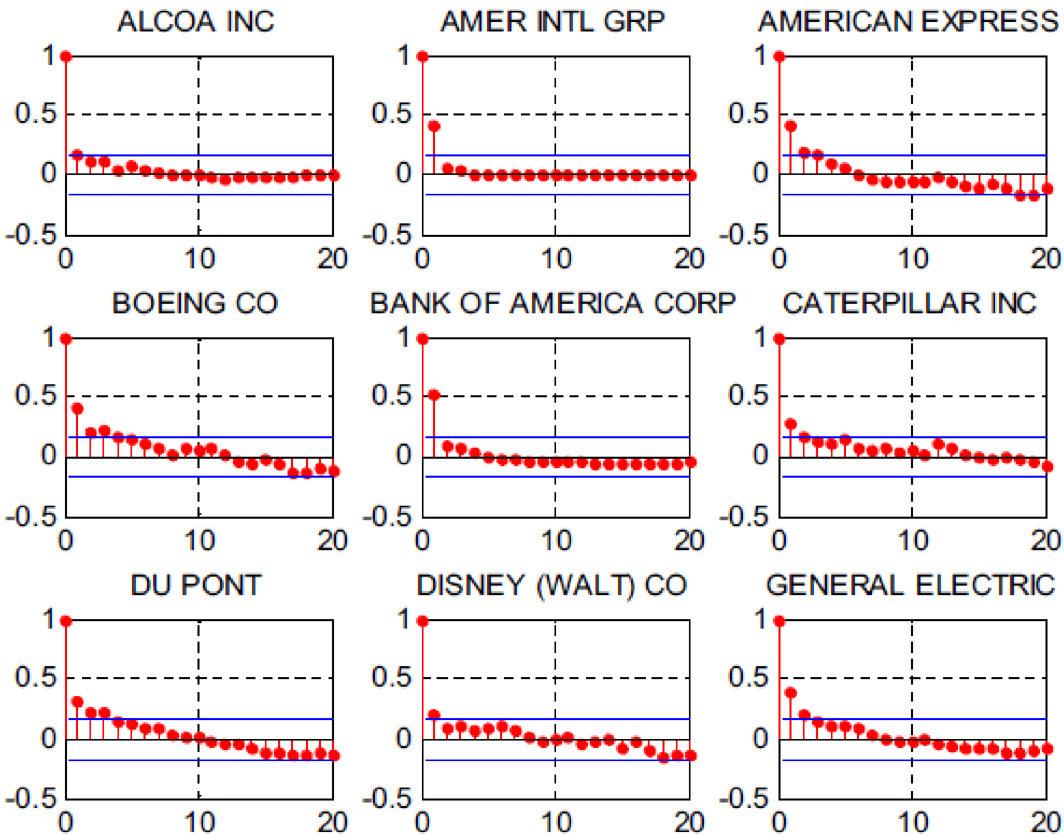


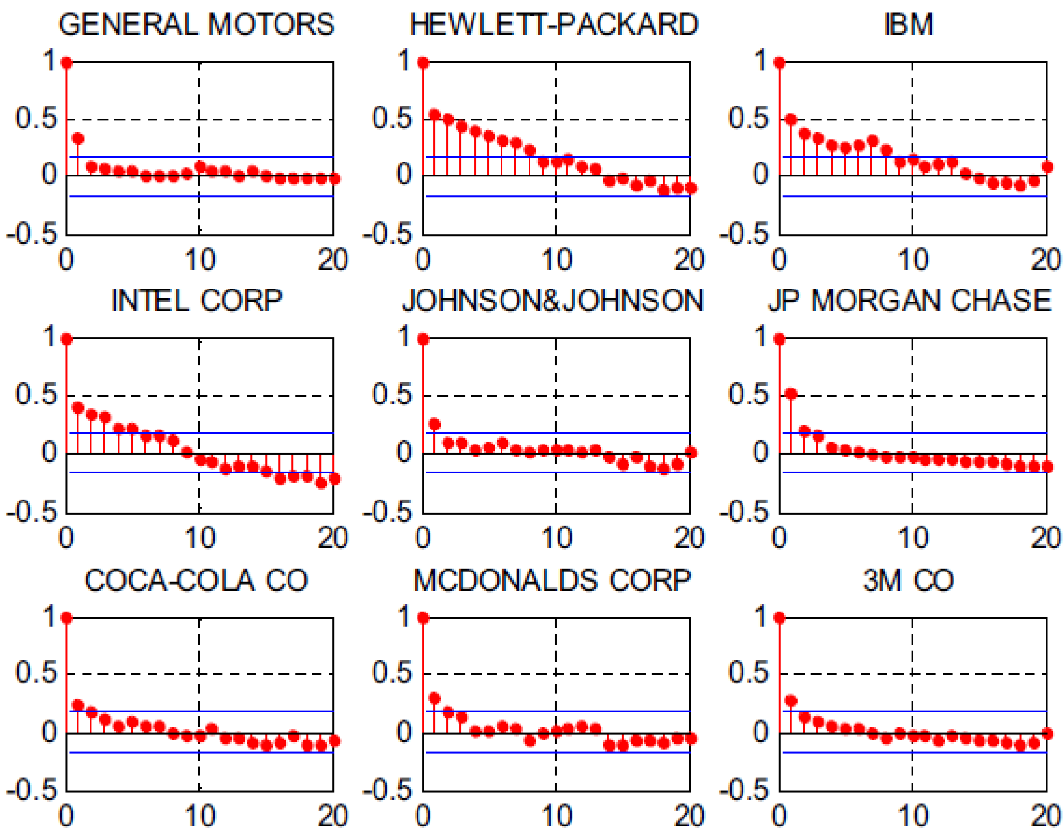

ALCOA INC
AMER INTL GRP
AMERICAN EXPRESS
BOEING CO
BANK OF AMERICA CORP
CATERPILLAR INC
DU PONT
DISNEY (WALT) CO
GENERAL ELECTRIC
Q4-97
Q1-03
Q2-08

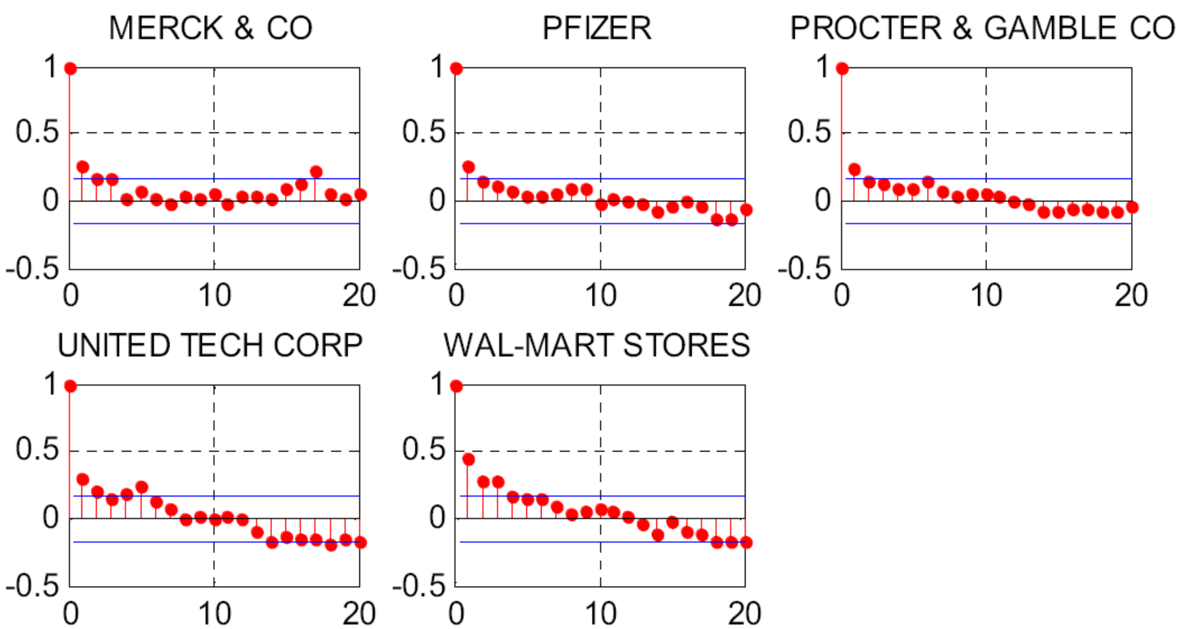


The solid line is the proxy for the true realized volatility which is computed from 30-minute returns and overnight returns. The dotted line is the realized volatility computed from daily returns. The sample covers the period from 1997 Q4 to 2008 Q2, 43 quarters in total. Both volatility measures are multiplied by 100.

Figure 3.2: Autocorrelation Functions for Logarithmic Quarterly Realized Volatility

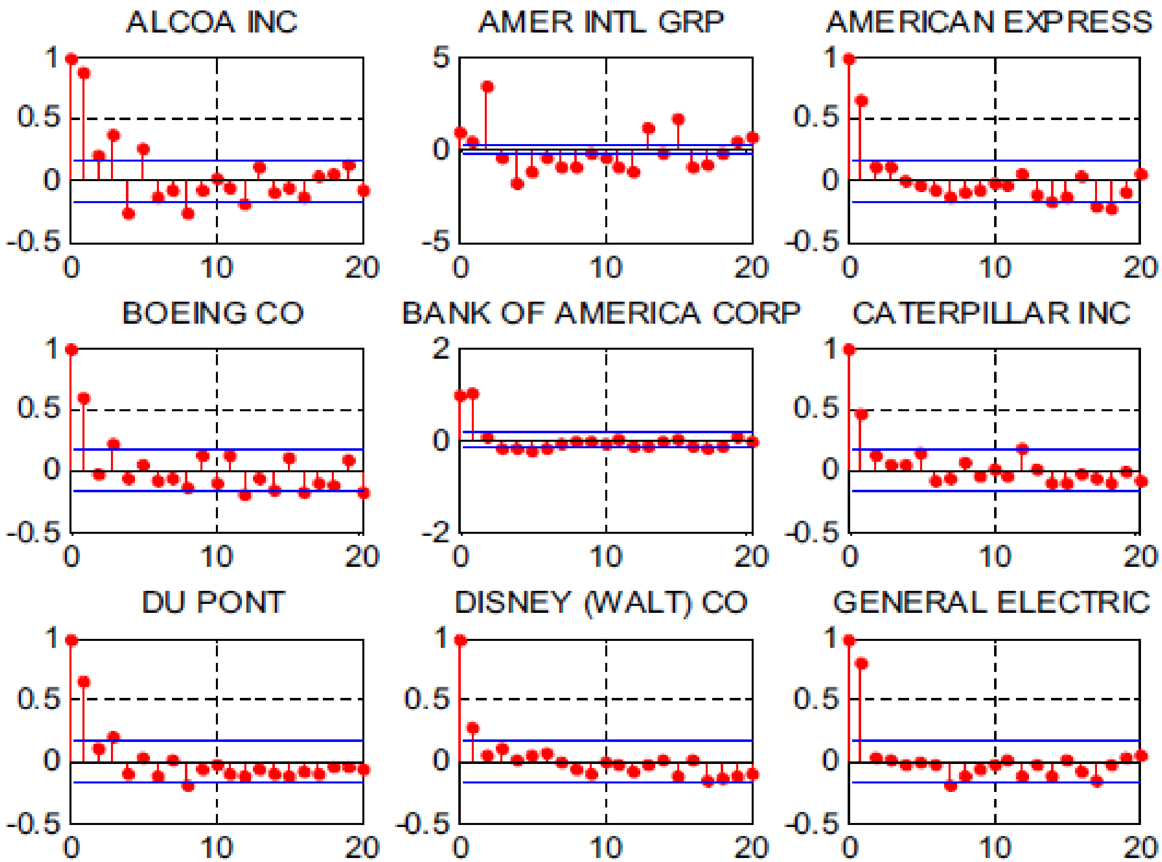


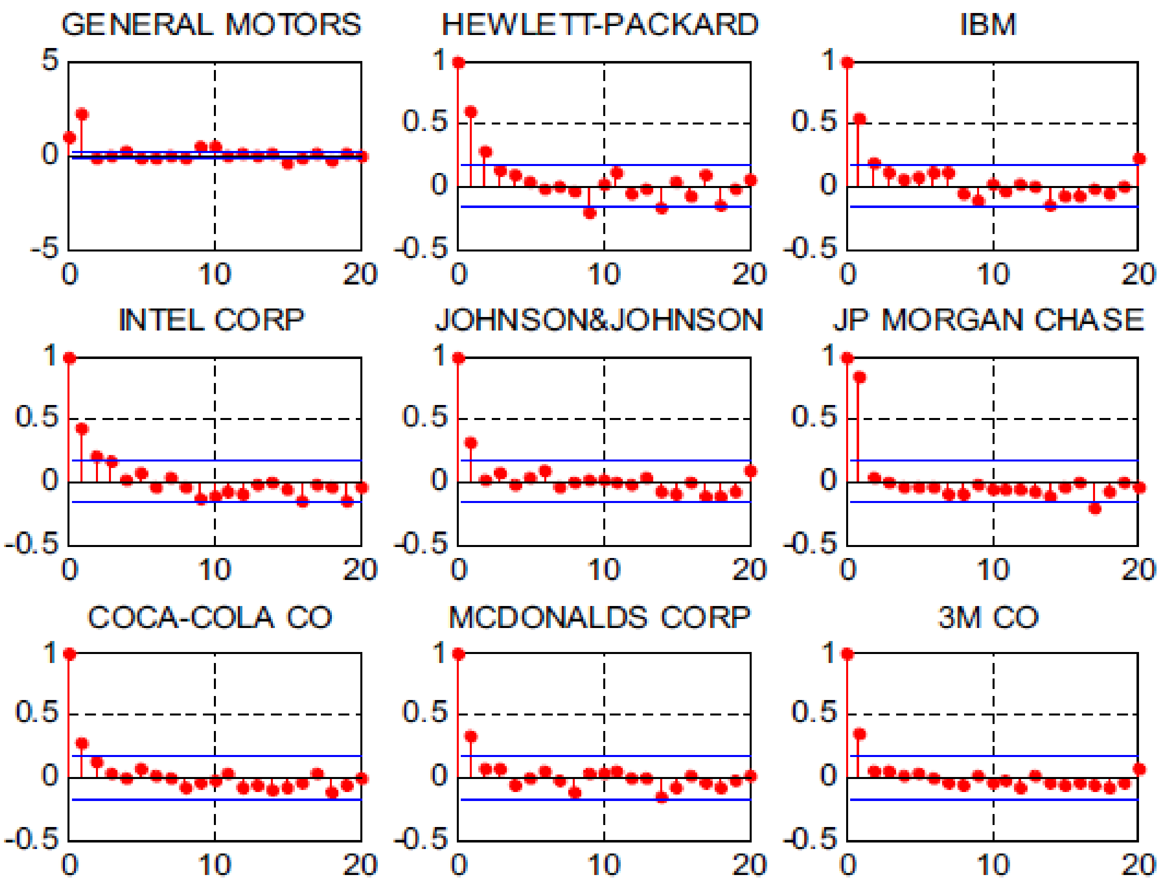

GENERAL MOTORS
HEWLETT-PACKARD
IBM
INTEL CORP
JOHNSON&JOHNSON
JP MORGAN CHASE
COCA-COLA CO
MCDONALDS CORP
3M CO
1
0.5
0
-0.5
0
10
20

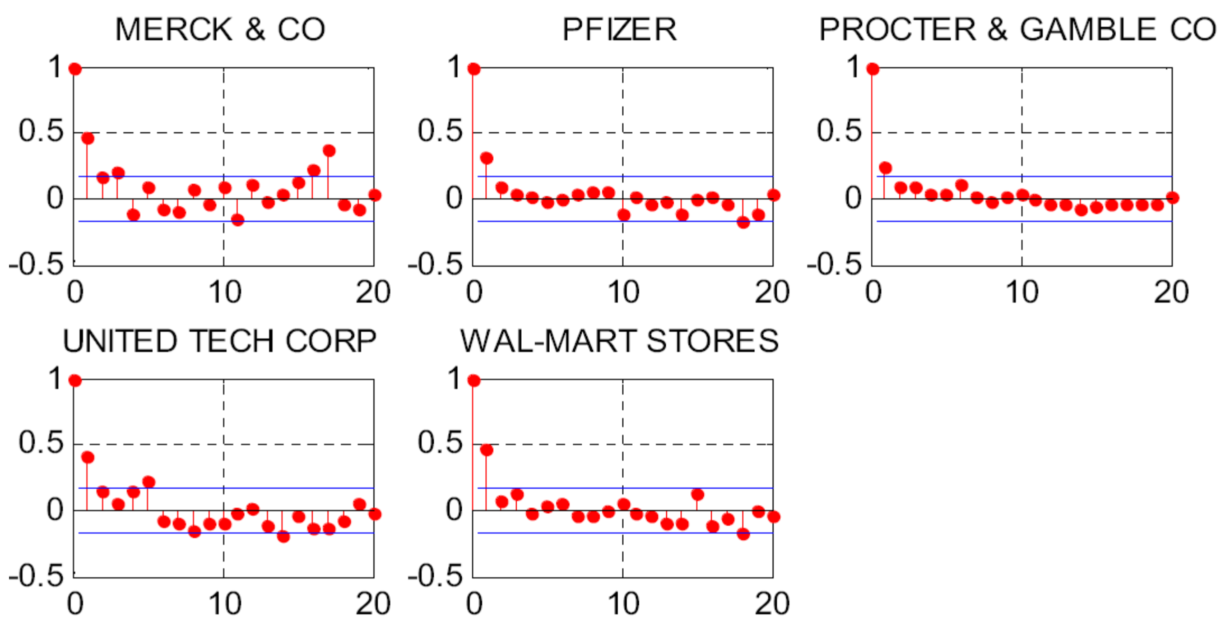


The quarterly realized volatility is computed from daily returns and the sample covers the period from 1997 Q4 to 2008 Q2. The marked confidence bands are at the 95% level.

Figure 3.3: Partial Autocorrelation Functions for Logarithmic Quarterly Realized Volatility




GENERAL MOTORS
HEWLETT-PACKARD
IBM
INTEL CORP
JOHNSON&JOHNSON
JP MORGAN CHASE
COCA-COLA CO
MCDONALDS CORP
3M CO



The quarterly realized volatility is computed from daily returns and the sample covers the period from 1997 Q4 to 2008 Q2. The marked confidence bands are at the 95% level.

Chapter 4

Quarterly Beta Estimation

4.1 Introduction

The recent advances in financial econometrics in realized variance and covariance measurement has introduced a new framework for the realized beta construction and modelling through the use of high-frequency data. A new beta measurement technique is introduced in Chen and Reeves (2009) who applied the Hodrick–Prescott filter (HP filter henceforth) with a parameter of 100 to monthly realized beta time series constructed from daily returns for 27 of the most liquid US stocks in the Dow Jones Industrial Average Index (DJIA). They found these HP filtered beta series followed the dynamics of the proxy of true realized beta formed from 30-minute returns over time and outperformed the industry standard beta measurement method from Fama and MacBeth (1973). The measurement errors are significantly reduced with this filter when compared with the true realized beta measures constructed from 30-minute intraday returns. This chapter extends the Chen and Reeves (2009) methodology to quarterly beta measurement frequency and confirms the usefulness of this technique. This technique will be used in Chapter 5 to construct the proxy of true beta time series over the long history. The portfolio manager can benefit by extracting beta from cheap daily data. Therefore, they can avoid the cost of purchasing high-frequency data and the computational burden of handling complicated high-frequency data.

The chapter is organized as follows. Section 4.2 describes the sample of data. The realized beta literature is briefly reviewed in Section 4.3. A brief review on the HP filter is presented in Section 4.4. Secion 4.5 discusses the empirical results and Section 4.6 concludes the chapter.

4.2 Data

To compare the Hodrick-Prescott filtered beta with other beta measures, we require stock return data on the intraday, daily and monthly basis. Monthly stock returns are used to estimate the Fama-MacBeth beta. Daily and 30-minute intraday returns are ingredients to construct realized beta at low and high frequencies respectively. We choose the 27 most liquid US stocks which are components of the Dow Jones Industrial Average index[1]. Both the daily and monthly stock return data are from the Center for Research in Security Prices (CRSP). The 30-minute intraday price data is obtained from Price-Data[2]. Furthermore, monthly, daily and 30-minute return data for the Dow Jones Industrial Average Index are also sourced from Price-Data. The sample period starts from October 1

[1]Three companies (Chevron Corp, Verizon Communication and Exxon Mobil Corp) are excluded due to insufficient data.

[2]www.grainmarketresearch.com

1997 and ends on July 31, 2008. The 30-minute intraday price is sampled from $9:30$AM to $3:30$PM. The price series is adjusted for dividend and stock splits.

4.3 Realized Beta Contruction

Barndorff-Nielsen and Shephard (2004b), Andersen et al. (2005) and Andersen et al. (2006) developed a theoretical framework to exploit the construction and modeling of realized beta. First, assume p_t denote logarithmic $N \times 1$ vector price process and suppose that p_t follows an Ito process,

$$dp_t = \mu_t dt + \sigma_t d\Omega_t \tag{4.1}$$

where μ_t is the $N \times 1$ drift vector, σ_t is the $N \times N$ diffusion matrix and Ω_t is the corresponding N-dimensional standard Brownian motion process. From the above model, the true and unobserved volatility is measured from 0 to T:

$$\int_0^T \sigma_t^2 dt \tag{4.2}$$

Let Δ be the sampling frequency and the interval of [0,T] could be divided into a number of the equal-spaced smaller intervals of Δ and $r_{i,t+\Delta,t} = p_{i,t+\Delta} - p_{i,t}$ be the compounded return over $(t, t+\Delta)$ for stock i. As the sampling frequency increases to infinity (i.e. $\Delta \to 0$), in theory the integrated volatility could be approximated as the sum of product of this finely-sampled high-frequency continuous returns, which could be displayed as the following model:

$$\int_0^T \sigma_t^2 dt \rightarrow \sum_{j=1,\ldots,[T/\Delta]} r_{t+j\cdot\delta} \cdot r_{t+j\cdot\delta} \tag{4.3}$$

The realized beta of a stock is defined as the ratio of the realized covariance of the stock and the market index to the realized variance of the market index. Following the above realized variance discussion, the realized covariance of stock i and the market M over the period [t,t+h] is

$$\widehat{\nu}_{iM,t,t+h} = \sum_{j=1,\ldots,[h/\Delta]} r_{i,t+j\cdot\Delta,\Delta} \cdot r_{M,t+j\cdot\Delta,\Delta} \tag{4.4}$$

Similiarly, the realized variance of the market over the period $[t, t+h]$ is:

$$\widehat{\nu}_{M,t,t+h} = \sum_{j=1,\ldots,[h/\Delta]} r^2_{M,t+j\cdot\Delta,\Delta} \tag{4.5}$$

Finally the realized beta of stock i is exhibited as follows:

$$\widehat{\beta}_{i,t,t+h} = \frac{\widehat{\nu}_{iM,t,t+h}}{\widehat{\nu}_{M,t,t+h}} = \frac{\sum_{j=1,\ldots,[h/\Delta]} r_{i,t+j\cdot\Delta,\Delta} \cdot r_{M,t+j\cdot\Delta,\Delta}}{\sum_{j=1,\ldots,[h/\Delta]} r^2_{M,t+j\cdot\Delta,\Delta}} \rightarrow \frac{\int_0^h \omega_{i,M,t+s} ds}{\int_0^h \omega_{M,t+s} ds} = \beta_{i,t,t+h} \tag{4.6}$$

Equation 4.6 shows the realized beta measure is a consistent measure of the true beta by sampling both stock return and market return at equally-spaced high frequency.

The realized beta measurement developed from continuous asymptotic distribution theory is based on the assumption of equally-spaced high frequency data. The market structure, trading activity and well-known microstructure issues have prohibited the sampling too frequent. Stocks listed in exchanges are only traded for a limited time period. Furthermore, the liquidity is a big issue for small stocks and the prices tend to jump discretely. Last but not least, bid/ask bounce, non-synchronous trading and other issues all create difficulty for the practical construction of realize beta. To circumvent these issues, DJIA stocks are well regarded as the most liquid stocks and we believe 30-minute sampling frequency is well enough to capture the information flow, mitigate common market microstructure problems and also lead to enough observations for quarterly realized beta estimation. Bollerslev et al. (2007) found that a sampling frequency of 22.5 minutes mitigates the effect of the microstructure noise for forty stocks in their data sample.

In this chapter, Δ in Equation 4.6 is specified as 30-minute, which is the length of each interval and the intraday price is sampled from 9 : 30AM to 3 : 30PM. Furthermore, T corresponds to one quarter. The purpose of this chapter to demonstrate how the investor can extract the same set of information from daily returns as that impounded in high-frequency intraday returns. This is of great interest to investors since daily data is available at almost no cost to investors. Our data sample is from 1997 Q4 to 2008 Q2, 43 quarters in total.

4.4 Hodrick–Prescott Filter

The HP filter used by Hodrick and Prescott (1997) has been widely used in empirical economic research to extract a smoothed non-linear trend from a time series at different frequencies. The theoretical framework for the HP filter can be summarized as follows: Let $\mathbf{y}' = (y_1, y_2, \cdots, y_N)$, be the observed series and assume that it can be decomposed into trend and cycle components. This implies $y_t = \tau_t + c_t$, with $\tau' = (\tau_1, \tau_2, \cdots, \tau_N)$ and $\mathbf{c}' = (c_1, c_2, \cdots, c_N)$. τ_t denotes the unobserved trend component and c_t the unobserved irregular component at time t. Given positive parameter, the estimated trend component

$\widehat{\tau_t}$ can be obtained by solving the following minimization problem:

$$\min_{|\tau|}\left[\sum_{t=1}^{N}(y_t-\tau_t)^2+\lambda\sum_{t=2}^{N-1}\left[(\tau_{t+1}-\tau_t)-(\tau_t-\tau_{t-1})\right]^2\right],\lambda>0 \tag{4.7}$$

The smoothing parameter λ in the HP filter is to control the smoothness of the trend component τ_t. The first term in the above equation is to minimize the variance in the noise component and the second is to reduce the variance in the growth rate of the trend component. The objective is to find an optimal λ for each stock in our sample that yields the minimum mean squared error (MSE) between the filtered trend component and the quarterly realized betas from 30-minute returns. To achieve this, a wide range of λ values from 1 to 300,000 with an interval of 50 are examined on the quarterly realized beta based on daily returns. The trend component derived from each run of λ is compared with our benchmark quarterly realized beta formed by 30-minute returns. The resulting MSE is computed as follows:

$$MSE=\frac{1}{n}\sum_{j=1}^{n}(\widetilde{\beta_{i,j}}-\widehat{\beta_{i,j}})^2 \tag{4.8}$$

where $\widetilde{\beta_{i,j}}$ represents the trend component for stock i in quarter j extracted for each λ when applying daily frequency data and $\widehat{\beta_i}$ denote the proxy for the true quarterly realized beta constructed by 30-minute returns in quarter j for stock i. n is the total number of quarters in our data sample, which is 43 quarters in total.

Finally, the optimal λ is determined for each stock that has the minimum MSE. Furthermore, the MSE values of unsmoothed betas are also calculated against true quarterly realized betas from 30-minute returns.

4.5 Results

The optimal λ for each stock is exhibited in Table 4.1. The results suggest that the optimal λ varies across stocks, ranging from 1 to 18501. The maximum reduction in MSE is 75.48% for McDonalds Corp, while the minimum reduction is 14.16% for Bank of America Corp. On average, the reduction in MSE based on the optimal smoothing is 50.55%. However, the high dispersion of optimal λ values does not offer a uniform solution to investors who are interested in a simple and practically applicable approach for a timely beta estimate.

One possibility to address this issue is to identify a fixed value of the smoothing parameter that is likely to bring substantial improvement in MSE across stocks. Chen and Reeves (2009) suggest an HP filter with $\lambda = 100$ (HP100 filter henceforth) and report comparable reduction in MSE to those with the optimal value of λ. Following this, our results confirm the validation of HP100 filter. Results in Column 4 in Table 4.1 suggest the majority of the reduction is preserved without imposing each individual optimal λ value. The improvements are almost unchanged for Boeing Corp, Disney (Walt) Corp, IBM, Intel Corp, Microsoft Corp and Procter & Gamble Corp and AT & T Inc. On average, the HP100 filter improves the accuracy by 46.51%, slightly lower than the individual optimal value. Overall, our results in Table 4.1 suggest that investors will benefit from the HP100 filtered beta, which provides a more timely and reliable beta estimate but also reduces the difficulty of estimation and removes noise when estimating quarterly beta.

Fama-MacBeth beta estimation has been widely used both in practice and academia. It is constructed by performing a 5-year rolling-window regression of monthly stock returns against monthly market returns. For example, the regression coefficient of past 5-year (60 months) stock returns from t-60 to t-1 on market returns is the beta estimate for current month t. Table 4.2 compares the performance of Fama-MacBeth beta, quarterly realized beta constructed from daily returns and the HP100 filtered beta. The average MSE for HP100 filtered beta is 0.0335, about half of daily-return-constructed beta and less than one-third of the Fama-MacBeth beta. Our results confirms the superiority of HP100 filtered beta to two other beta measures.

Figure 4.1 visually demonstrates that the HP100 filtered beta uncovers the smooth trend from original quarterly realized beta obtained from daily returns. The HP100 filtered beta and quarterly realized beta based on intraday returns are plotted in Figure 4.2. The HP100 filtered beta, which is essentially the original beta series excluding noise, follows very closely to the benchmark quarterly realized beta. As demonstrated in Chen and Reeves (2009), the HP100 filter beta extracts the most relevant information and reduces the measurement error. In sharp contrast, the Fama-MacBeth beta is consistently above the benchmark quarterly realized beta based on 30-minute returns in Figure 4.3, failing to identify the dynamics of the benchmark quarterly realized beta. The Fama-MacBeth beta fails to identify the uptrend in the benchmark beta series for stocks during the global financial crisis, such as JPMorgan Chase, General Motors, American Express, while the HP100 filtered betas in Figure 4.2 move up and down accordingly with the benchmark quarterly beta. Therefore, the HP100 filter is a valid and useful tool for market participants to extract accurate quarterly beta estimates from daily returns.

4.6 Conclusion

Following Chen and Reeves (2009), this chapter applies the HP filter with a uniform parameter of 100 to construct the quarterly realized beta using daily returns. The superiority of this HP100 filtered beta estimation approach, claimed in Chen and Reeves (2009) for monthly beta measurement, is found in this chapter as well for quarterly beta measurement. Our results suggest that the filtered quarterly beta extracts the smoothly changing component and tracks closely to the benchmarked realized beta based on 30-minute stock returns. Importantly, the average measurement error of the HP100 filtered beta is significantly lower than other beta estimation approaches.

Table 4.1: Reduction in MSE by Applying HP Filter

Company Name	Optimal λ	%MSE is Reduced when Using λ	%MSE is Reduced When $\lambda = 100$
ALCOA INC	13901	51.44%	43.51%
AMER INTL GRP	51	35.23%	28.65%
AMERICAN EXPRESS	1	20.30%	9.90%
BOEING CO	101	53.02%	53.02%
BANK OF AMERICA CORP	701	14.16%	7.87%
CITIGROUP INC	1	21.02%	6.18%
CATERPILLAR INC	1	40.93%	36.09%
DU PONT	1	52.52%	38.85%
DISNEY (WALT) CO	51	61.20%	60.76%
GENERAL ELECTRIC	1	47.11%	45.04%
GENERAL MOTORS	1	48.96%	46.50%
HOME DEPOT INC	1201	49.48%	44.87%
HEWLETT-PACKARD	51	71.87%	70.42%
IBM	151	71.02%	71.02%
INTEL CORP	201	37.66%	37.02%
JOHNSON&JOHNSON	51	44.13%	42.96%
JP MORGAN CHASE	1	14.32%	10.84%
COCA-COLA CO	401	45.56%	44.76%
MCDONALDS CORP	401	75.48%	74.79%
3M CO	51	67.08%	65.23%
MERCK & CO	14001	68.27%	61.30%
MICROSOFT CORP	101	73.72%	73.72%
PFIZERa	351	74.78%	73.20%
PROCTER & GAMBLE CO	151	68.58%	68.41%
A T & T INC	151	60.07%	59.84%
UNITED TECH CORP	3251	41.93%	36.84%
WAL-MART STORES	18501	54.98%	44.28%
Average MSE Reduction due to smoothing		50.55%	46.51%
MAX MSE Reduction due to smoothing		75.48%	74.79%
Min MSE Reduction due to smoothing		14.16%	6.18%

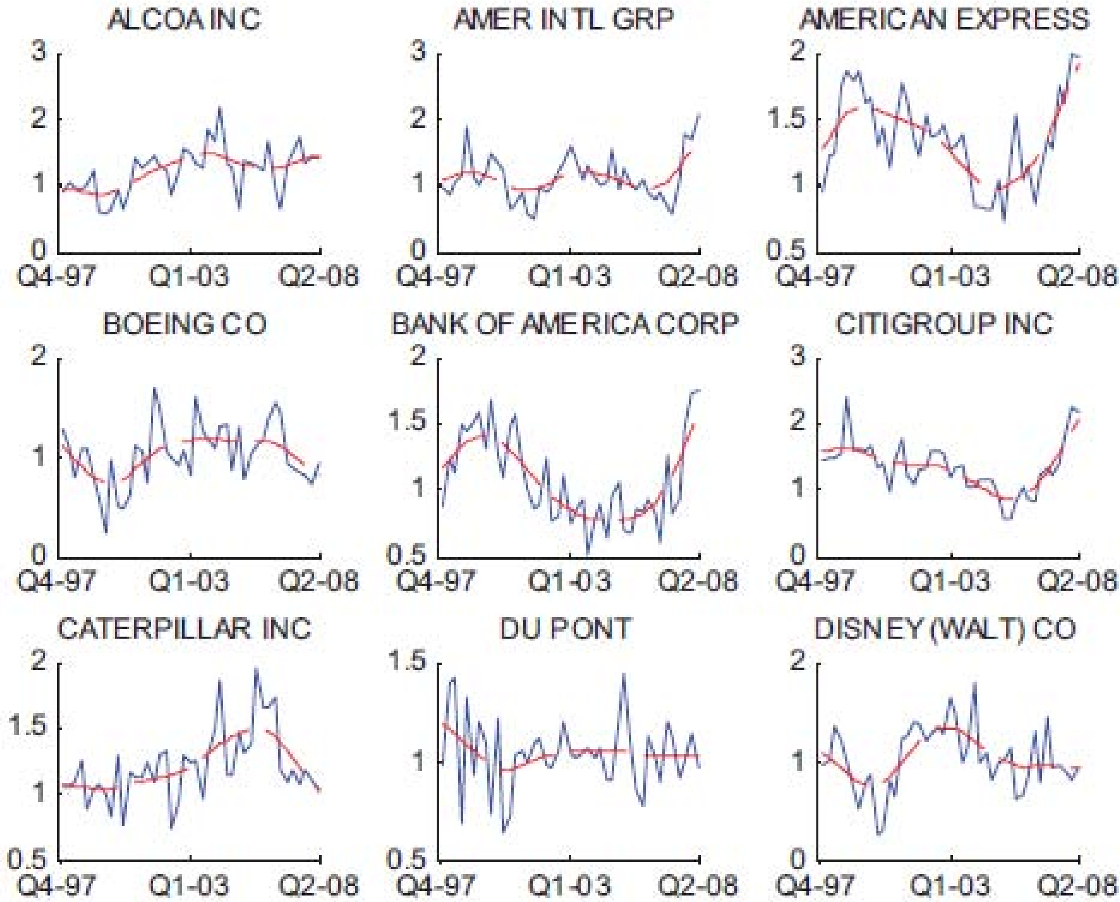
Note: The data sample ranges from Q4 1997 to Q2 2008. HP filter is applied to quarterly realized beta formed by daily stock returns and smoothing parameter (λ) changes from 1 to 300,000 with an interval of 50. The optimal λ is reported in Column 2 which leads to the smallest MSE when comparing trend component to the benchmark quarterly realized beta formed by 30-minute stock returns. The MSE reduction is reported in Column 3 due to optimal λ. Column 4 reports the MSE reduction by fixing λ equals 100.

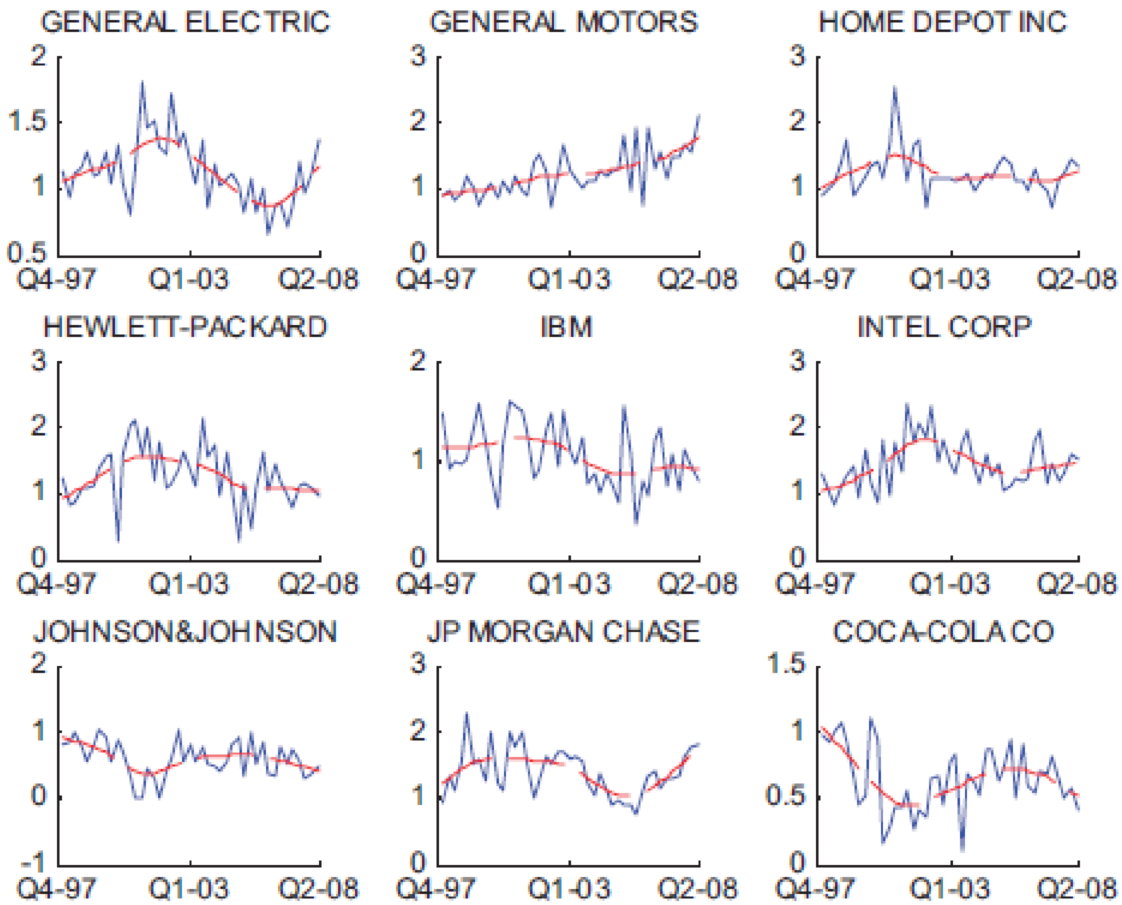
Table 4.2: MSE of Different Beta Measures Benchmarked against Quarterly Realized Betas from 30-minute Returns

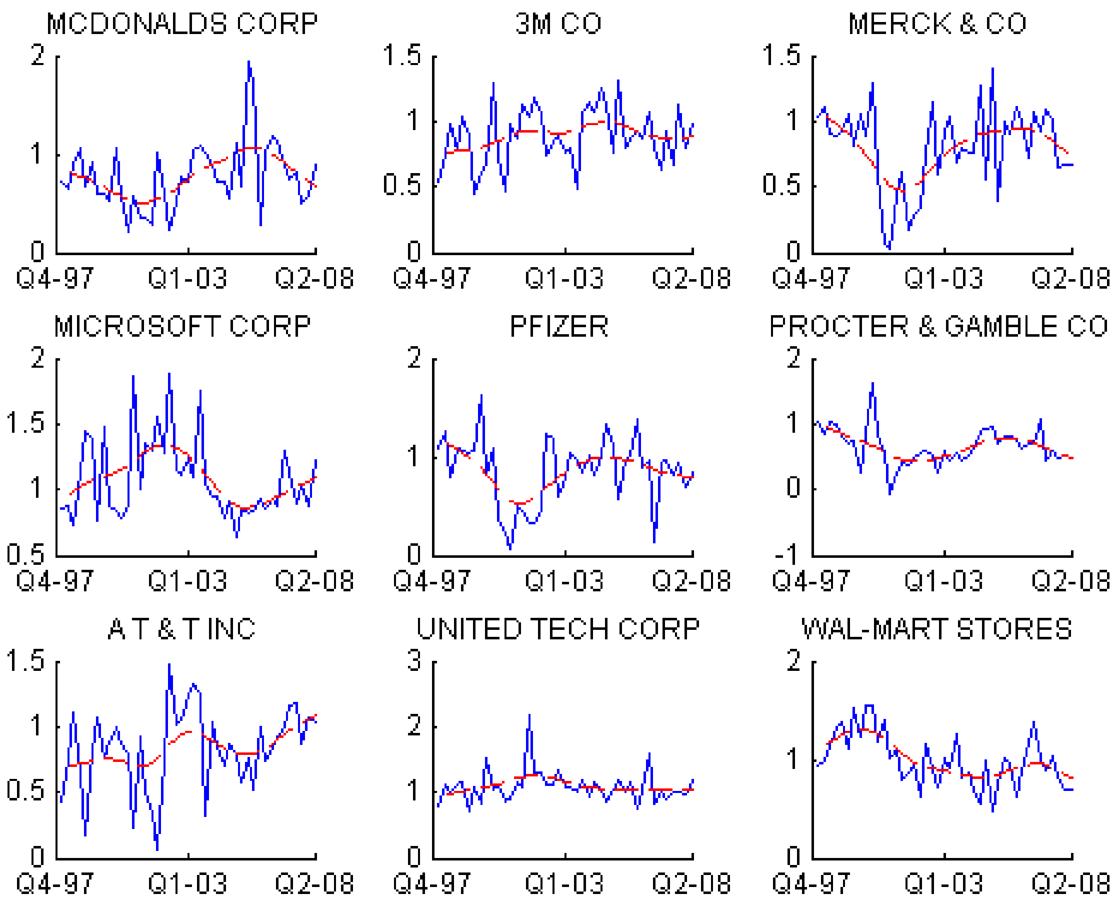
Company Name	Fama-MacBeth Beta by Monthly Returns	Quarterly Realized Beta by Daily Returns	HP 100 Beta
ALCOA INC	0.3010	0.1009	0.0570
AMER INTL GRP	0.0975	0.0684	0.0488
AMERICAN EXPRESS	0.0832	0.0606	0.0546
BOEING CO	0.0921	0.0645	0.0303
BANK OF AMERICA CORP	0.1185	0.0445	0.0410
CITIGROUP INC	0.1535	0.0566	0.0531
CATERPILLAR INC	0.0472	0.0496	0.0317
DU PONT	0.0219	0.0278	0.0170
DISNEY (WALT) CO	0.0713	0.0683	0.0268
GENERAL ELECTRIC	0.0394	0.0242	0.0133
GENERAL MOTORS	0.0507	0.0815	0.0436
HOME DEPOT INC	0.0909	0.1063	0.0586
HEWLETT-PACKARD	0.2067	0.1109	0.0328
IBM	0.1645	0.0597	0.0173
INTEL CORP	0.2999	0.0786	0.0495
JOHNSON&JOHNSON	0.0909	0.0426	0.0243
JP MORGAN CHASE	0.1585	0.0775	0.0691
COCA-COLA CO	0.0807	0.0496	0.0274
MCDONALDS CORP	0.0912	0.0730	0.0184
3M CO	0.0248	0.0325	0.0113
MERCK & CO	0.0821	0.0602	0.0233
MICROSOFT CORP	0.0557	0.0586	0.0154
PFIZER	0.1074	0.0694	0.0186
PROCTER & GAMBLE CO	0.1603	0.0592	0.0187
A T & T INC	0.1270	0.0884	0.0355
UNITED TECH CORP	0.1383	0.0570	0.0360
WAL-MART STORES	0.1600	0.0542	0.0302
Average MSE	0.1154	0.0639	0.0335

Note: The data sample covers over the period from Q4 1997 to Q2 2008. Three beta measures are compared with quarterly benchmark realized beta which is constructed from 30-minute stock returns. The first one is Fama-MacBeth beta. The second measure is formed by daily returns. The third candidate is the trend component by applying HP filter with λ=100 to quarterly realized beta by daily returns.

Figure 4.1: Plot of HP 100 Filtered Quarterly Realized Betas, Formed by Daily Returns

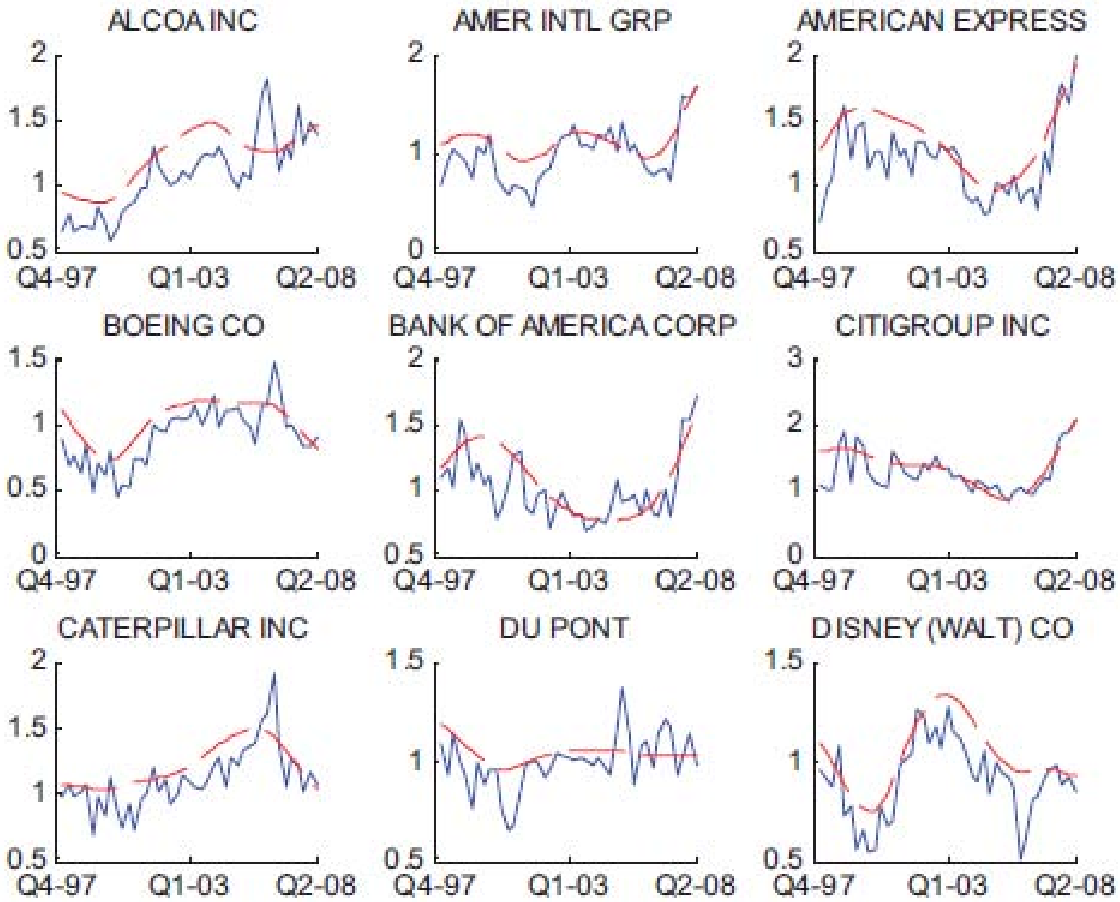


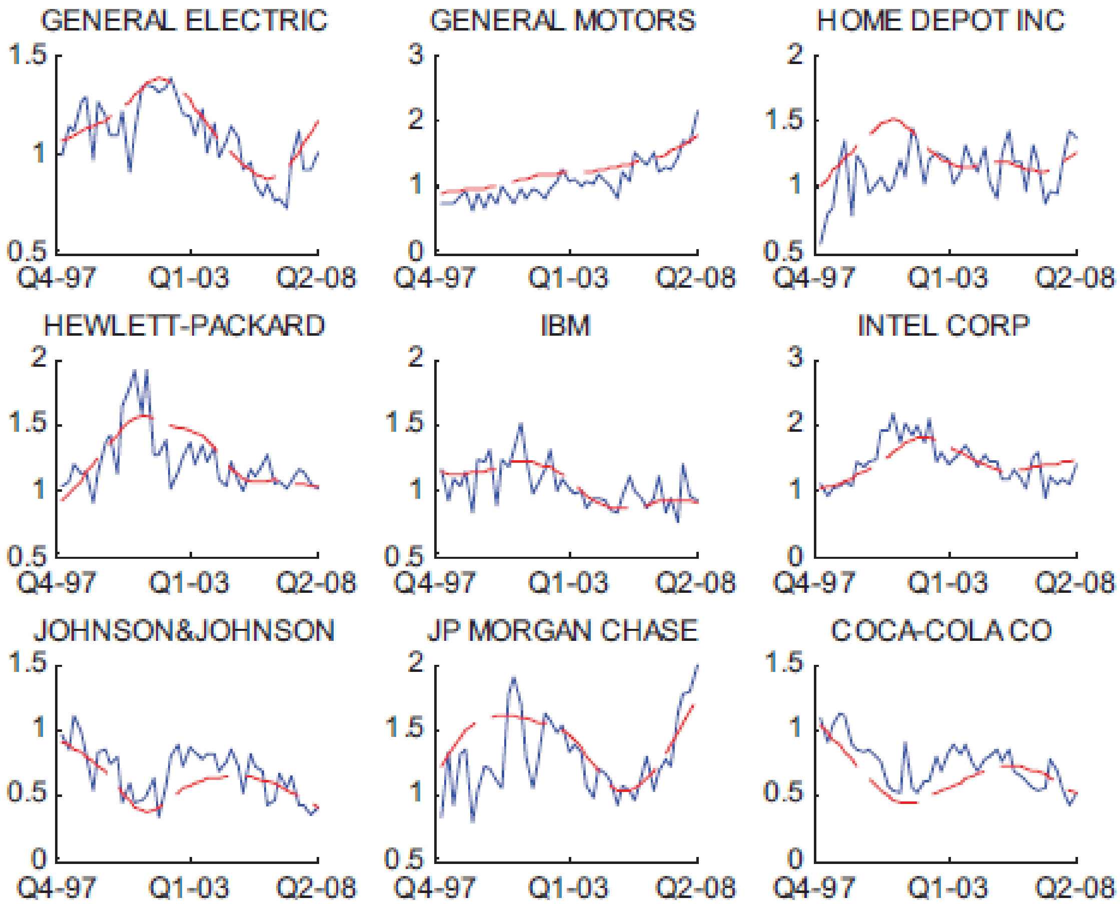

GENERAL ELECTRIC
GENERAL MOTORS
HOME DEPOT INC
HEWLETT-PACKARD
IBM
INTEL CORP
JOHNSON&JOHNSON
JP MORGAN CHASE
COCA-COLA CO
Q4-97
Q1-03
Q2-08



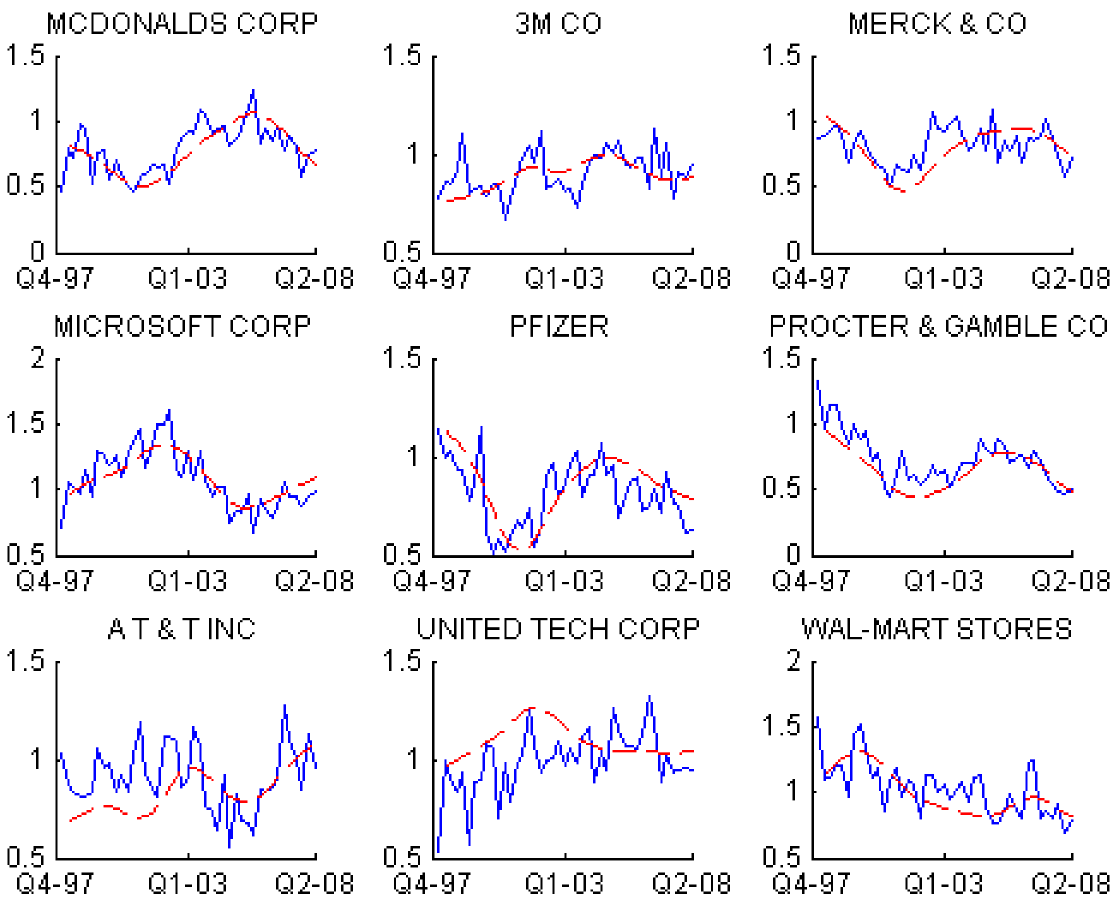
The quarterly realized beta is formed by the daily returns and the smoothed beta is the trend component extracted by applying HP filter with $\lambda = 100$. The sample is from Q4 1997 to Q2 2008, 43 quarters in total.

Figure 4.2: HP 100 betas versus Benchmark Quarterly Realized Betas



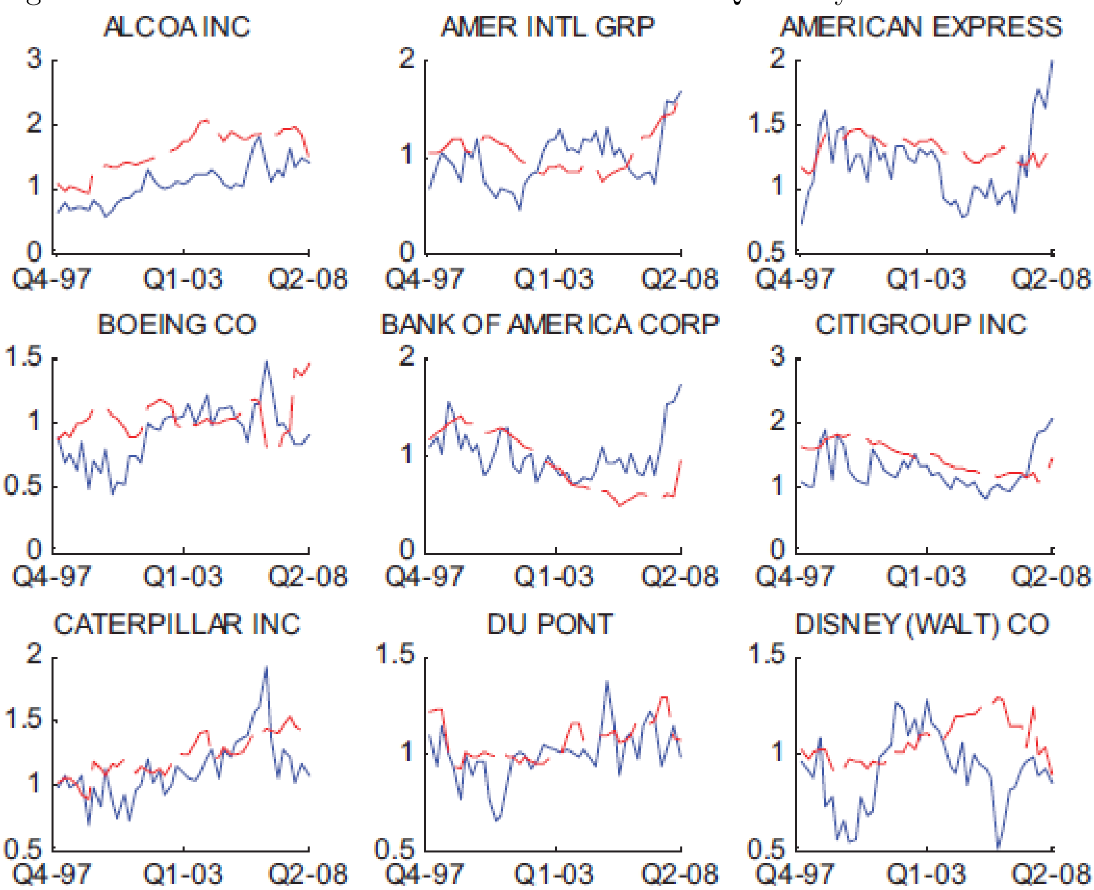


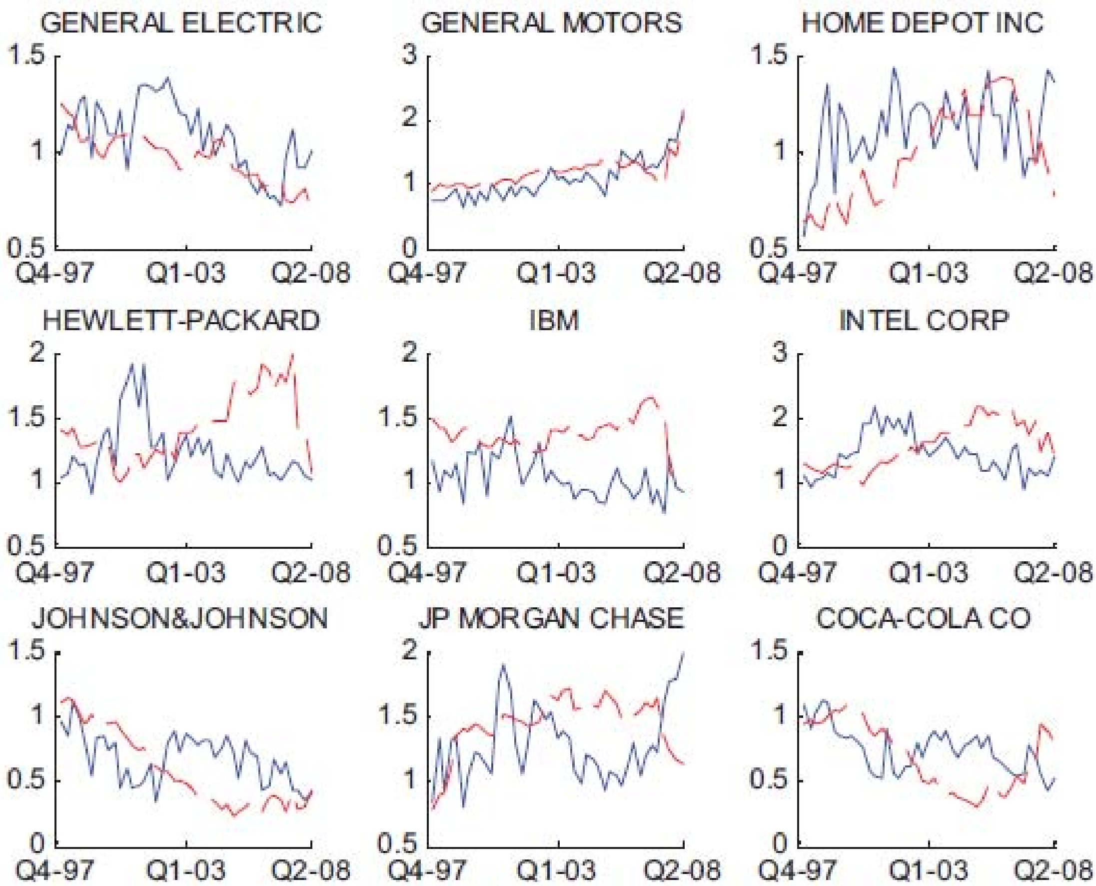
GENERAL ELECTRIC
GENERAL MOTORS
HOME DEPOT INC
HEWLETT-PACKARD
IBM
INTEL CORP
JOHNSON&JOHNSON
JP MORGAN CHASE
COCA-COLA CO
Q4-97
Q1-03
Q2-08



The benchmark quarterly realized beta is plotted solid line and constructed by 30-minute returns. The HP100 beta is plotted in dotted line and is the trend component extracted by applying HP filter with $\lambda = 100$ to quarterly realized beta by daily returns. The sample is from Q4 1997 to Q2 2008, 43 quarters in total.

Figure 4.3: Fama-MacBeth betas versus Benchmark Quarterly Realized Betas




GENERAL ELECTRIC
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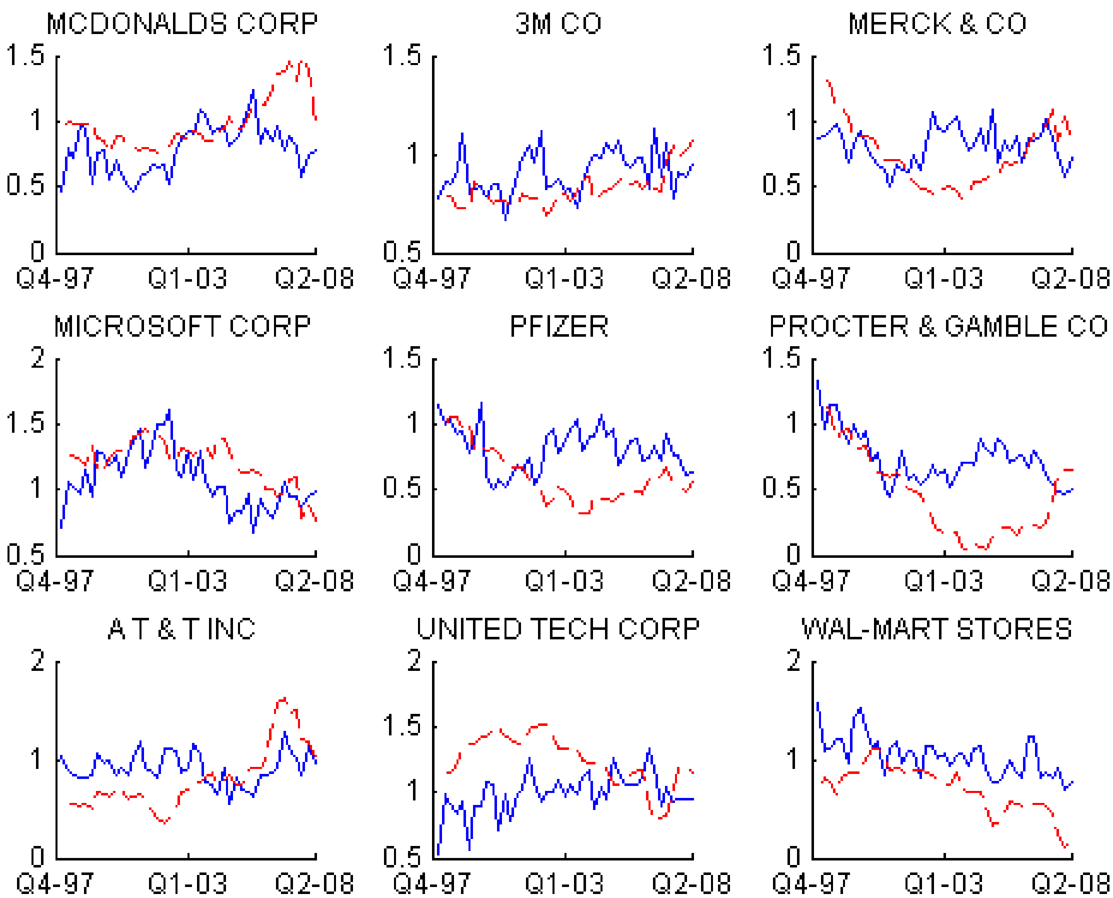
JP MORGAN CHASE
COCA-COLA CO
Q4-97
Q1-03
Q2-08



The benchmark quarterly realized beta is plotted solid line and constructed by 30-minute returns . The Fama-MacBeth beta is plotted in dotted line. The sample is from Q4 1997 to Q2 2008, 43 quarters in total.

Chapter 5

Betas, Hedge Funds and the Myth of Market Neutrality

5.1 Introduction

The tremendous growth of the hedge fund industry over the last ten years has brought the alternative investment industry very much into the mainstream. From an estimated \$39 billion under management in 1990, the hedge fund industry as of the end 2008 totalled 10,000 funds globally with over \$1.9 trillion USD in assets under management. The demand for alternative investments has not been limited to the traditional clientele of high net worth investors. In fact, the growth has largely been fuelled by demand from pension funds, sovereign funds and especially financial institutions. The catalyst for this significant allocation to hedge funds has been the promise of strong absolute returns and low correlation with traditional portfolio holdings. The growth in assets under management has not, however, been uniform across hedge fund styles. Certain hedge fund strategies, such as convertible arbitrage and merger arbitrage, face capacity constraints and diminishing returns to scale. The most significant benefactors of this insatiable demand for alternative funds have been the Long/Short equity and equity market neutral funds. Although the long/short equity funds choose to be directional and attempt to time the market using either systematic or discretionary rules, the equity market neutral funds should exhibit returns that are unaffected by fluctuations in the overall market.

The underlying principle behind equity market neutral funds is pretty simple. The managers aim to construct a portfolio that combines both long and short positions such that the overall portfolio has no exposure to market risk. The criteria for selecting the long and short positions can vary substantially from one manager to another however the ultimate goal is the same for all equity market neutral funds. Given the supposed high level of sophistication and technical know-how that hedge fund managers are expected to exhibit (this is why we pay them 2/20 after all), it is quite troubling to observe how many of these market neutral funds do not deliver true beta neutrality. In fact, Patton (2009) finds that over 25% of self-declared market neutral funds exhibit significant market exposure. The recent financial crisis provided further evidence of significant beta exposure by equity market neutral hedge fund managers as over 70% of funds reporting to Hedge Fund Research (HFR) finished 2008 in the red and the HFR Equity Market Neutral sub-index finished the year down nearly 10%. The question that we must ask ourselves therefore is whether these supposedly skilled managers are purposely straying from their original strategy hoping to enhance their returns with a bit of beta exposure, or is there perhaps a fundamental flaw in the way they are constructing their portfolios and estimating their beta exposures.

The construction of a truly market neutral portfolio depends inherently on the ability of the manager to accurately measure and forecast the beta exposure of his long and short portfolios. The greater the estimation error of the betas, the more likely the fund is to have a significant residual beta exposure and, therefore, the greater the potential exposure to systematic risk factors. This, of course, is precisely what investors hope to avoid when investing in equity market neutral funds. They do not want to be paying alpha fees for beta returns.

In this chapter, we build on recent literature that has highlighted some of the significant advantages of using higher frequency data to calculate and forecast the beta of stocks (Barndorff-Nielsen and Shephard (2004b), Andersen et al. (2005 and 2006), Ghysels and Jacquier (2006) and Hooper et al. (2008)). We construct a momentum-based market (beta) neutral equity portfolio using stocks comprising the S&P 100 index, and demonstrate the impact that the beta estimation approach has on the ex-post beta neutrality of the fund. We find that using daily data to evaluate realized betas allows us to better capture the true market exposure of the portfolio. In turn, this leads to market neutral portfolios that have substantially less unwanted systematic risk exposure. We believe that the inability of equity market neutral funds to exhibit market neutrality in their performance can, in large part, be attributed to the fact that they use out-dated and inaccurate beta estimation techniques when constructing their portfolios.

The rest of the chapter is organized as follows. Section 5.2 provides some background on realized beta. Section 5.3 describes the methodology. Secion 5.4 and 5.5 present the data and results respectively. Our conclusion are presented in section 5.6.

5.2 Realized beta

An in-depth literature review on realized beta construction is provided in Seciton 4.3. The realized beta measure is consistent for the true beta by sampling both security return and market return at an ultra high frequency.

5.2.1 Hodrick–Prescott filter and HP filtered realized beta

A detailed literature review on Hodrick–Prescott filter is provided in Seciton 4.4. Chen and Reeves (2009) experimented with a wide range of λ values finding 100 to be suitable for filtering monthly realized betas constructed from daily returns. These HP100 betas yielded on average close to the smallest measurement error when evaluated against the monthly realized betas constructed from 30-minute returns. Their results were robust across their entire sample of Dow Jone stocks. Hence, following the same methodology, we examined the same sample of stocks at the quarterly frequency. That is, we constructed quarterly realized betas from daily returns and experimented with a wide range of λ values also finding 100 to yield on average close to the smallest measurement error when evaluated against the quarterly realized betas constructed from 30-minute returns. Thus in this study, we measure ex-post quarterly betas by computing quarterly realized betas from daily returns which are smoothed to remove measurement error with the HP100 filter.

5.3 Methodology

There are three steps to the methodology. First, we will construct a dollar neutral long/short portfolio, using price momentum of stocks listed in the S&P 100 index. Next, we estimate the beta of the overall portfolio using four different approaches in order to calculate the market exposure (residual beta) of the portfolio. This residual beta will be hedged using futures contracts on the S&P 500 index in order to make the portfolios beta neutral. Finally, we will calculate the ex-post beta of our "market neutral" portfolios in order to ascertain which measure of beta allowed us to generate the most "ex-post" beta-neutral portfolio.

5.3.1 Beta forecasting models

To evaluate which beta forecasting model yields the best *ex post* beta neutral strategy, four candidate models are used in this chapter. These four models are selected based on their popularity and/or econometric justification.

Fama-MacBeth Beta

The Fama-MacBeth (1973) beta, which is the regression slope coefficient from 5 years of monthly stock returns regressed onto a constant and the market returns, is the most widely used beta estimate by both academics and practitioners. This beta forecast is regarded as the industry standard.

Quarterly realized beta

To calculate the quarterly realized beta we apply the methodology described in Andersen et al. (2006), and presented in section 5.2. As demonstrated by Andersen et al. (2006), the daily return sampling frequency may be used to construct quarterly realized betas. If we refer back to equation 4.6, this means that h will be quarterly and delta will be daily. Ghysels (1998) shows that constant beta models have outperformed more sophisticated models of time-varying beta. Thus, this random walk model is used as one of our beta forecast candidates. The realized beta is calculated each month by utilizing the previous three months of daily stock and market returns.

Autoregressive Quarterly realized beta

Hooper et al. (2008) evaluated a number of competing forecasting models that used quarterly realized betas. The following equation is used for the autoregressive model with p lags:

$$\beta_{i,t+1} = \alpha_{i,0} + \alpha_{i,1}\beta_{i,t} + \alpha_{i,2}\beta_{i,t-1} + \ldots + \alpha_{i,p}\beta_{i,t-(p-1)} + \epsilon_{i,t+1} \tag{5.1}$$

Hooper et al. (2008) determine that an AR(1) model provides the most accurate forecasts if only 5 years of data is available. Following this result, the third beta candidate is the AR(1) quarterly realized beta forecast using the past 20 quarters of realized betas. This is calculated in two steps: Firstly, the non-overlapping three-month realized beta series is constructed from daily stock and S&P 500 index returns over the 5 year period. The second step is to fit the AR(1) model to the constructed realized beta time series and produce the one step ahead (three month) forecast.

Annual realized beta

Reeves and Wu (2010) evaluate the forecasting performance of constant beta models over short horizons, relative to the recently suggested autoregressive models of quarterly realized betas and find that a constant beta model computed from daily returns over the last 12 months generates the most accurate quarterly forecast of beta. Therefore, we construct our fourth beta candidate by computing a realized beta from daily returns over the previous year. If we again refer back to equation 4.6, this means that h will be annual and delta will be daily.

5.3.2 The Momentum Portfolio Construction

Price momentum is widely used by portfolio managers as a buy or sell signal for stocks, and empirical research has sanctioned this type of strategy by finding that there appears to be a risk premia associated to momentum strategies (see Jegadeesh and Titman (1993) and Carhart (1997)). In fact, the momentum factor has become commonplace in most equity asset pricing models. Strategies that buy stocks that have performed well in the past and sell stocks that have performed poorly in the past generate significant positive returns over 3- to 12-month holding periods. The profitability of these strategies are not due to their systematic risk or to delayed stock price reactions to common factors. The evidence is consistent with delayed price reactions to firm-specific information.

A strategy that selects stocks on the basis of returns over the past J months and holds them for K months (a J-K strategy), as discussed in Jegadeesh and Titman (1993) and Carhart (1997), is constructed as follows:

- At the beginning of each month t, the stocks are ranked in ascending order on the basis of their returns in the past J months
- Based on these rankings, form ten equal-weighted decile portfolios
- In each month t, the strategy buys the winner portfolio and sells the loser portfolio and holds this position for K months
- The strategy closes out the position in month t+K

For the purpose of our chapter, we assume that the holding period, K, is three months and that the ranking period, J, is 11 months. Following the literature , we skip a month between the portfolio formation period and the holding period. The gap avoids the short term reversals which might contaminate the momentum strategy. Let $r_{i,t}$ denote return of stock i in month t. The cumulative return for stock i over the months from $t-12$ to $t-1$ is calculated as $\prod_{s=t-12}^{s=t-2}(1+r_{i,s})-1$. One Quarter in this chapter means rolling three-month.

S&P 100 index stock components are used as the sample and returns on S&P 500 index is used as the broad market return. We re-construct the actual S&P 100 index, adjusting the index for all additions and deletions. (The complete list of changes to the index is provided in Table 3). In this way, we ensure our portfolio construction process is realistic and there is no survivorship bias. The portfolio construction protocol is to rank the 100 component stocks on the basis of their cumulative returns over the 11-month formation period, that is $\prod_{s=t-12}^{s=t-2}(1+r_{i,s})-1$. We identify the top and bottom deciles, and enter into equal-weighted long positions in the 10 stocks in the top decile and enter an equal-weighted short positions in the 10 stocks in the bottom decile.

Since our holding period, K, is three months we need to ensure that results are not sensitive to the choice of starting month. In order to overcome this possible bias, we run 3 portfolios with overlapping holding period. At the end of any given month t, one of the three portfolios will reach the end of its' holding period, and a new portfolio will need to be formed. This newly formed portfolio, consisting of a \$100 million long position and \$100 million short position, is dollar neutral but not necessarily beta neutral. Our objective, however, is to construct beta neutral portfolio; therefore, the residual market exposure of the long/short portfolio is hedged using S&P 500 E-mini futures contracts[1]. The number of contracts is calculated as $10,000,000(\sum_{i=1}^{10} \beta_{l,i,t} - \sum_{j=1}^{10} \beta_{s,j,t})/P_t$, where $\beta_{l,i,t}$ is the beta forecast for long stock i at time t, $\beta_{s,j,t}$ is the beta forecast for short stock j at time t and P_t is futures price at time t.

5.3.3 Ex-post Beta analysis

By construction, the portfolio, consisting of long & short positions and overlay futures hedge, is beta neutral based on beta forecasts. The key to measure the accuracy of beta forecasts is to evaluate our *ex post* beta on the momentum portfolio. As stated before, there are three portfolios in each month and their *ex post* performance are treated separately. Accordingly, there are three sets of ex post betas for stocks. That is, the first set is comprised of 64 periods, including Jan 1993- Mar 1993, Apr 1993 - Jun 1993, . . . , Oct 2008 - Dec 2008. The second set has 63 periods over Feb-Apr 1993, May - Jul 1993, . . . , Aug-Oct 2008. The third set covers 63 periods as well, including Mar-May 1993, Jun-Aug 1993, . . . , Sep-Nov 2008. HP100 filter is applied to each realized beta series in three different time frame sets. This filtered tri-monthly beta series is our benchmarking true beta series.

[1]Detail about S&P 500 E-Mini futures contracts is on www.cmegroup.com

There are two ways to evaluate ex-post beta. First, equal-weights are assumed to evaluate the ex-post beta of the momentum portfolio at the end of the quarter. This weight scheme is motivated by the fact that equal weight is assigned to each stock when constructing the momentum portfolio. Consequently, it isolates the effect of the changing weights of stocks in the portfolio, and attributes the ex post portfolio to our accuracy of beta forecasts. Second, realized daily return is compounded to calculate the realized weight of each stock in the portfolio. The snapshot of stock weights at the end of holding period captures the variation of weights resulting from the stock specific performance over the holding period.

5.4 Data

S&P 100 index components are considered as our stock universe. We choose the S&P 100 index because it is composed of the most liquid stocks with the largest market capitalization in the US. Furthermore, futures on S&P 500 index is the most liquid futures contract and traded almost around the clock.

Both monthly and daily data for the S&P 100 index stock components and S&P 500 index are obtained from the Center for Research in Security Prices $CRSP$. The data set covers from 4 Jan 1988 to 31 Dec 2008. The monthly S&P 500 E–Mini Futures price data is sourced from datastream from 31 Jan 1992 to 31 Dec 2008.

The hourly intraday price data of current and historical stock components in the S&P

100 index is obtained from Price-Data and TAQ database. High frequency data sample ranges from January 2, 2003 to December 31, 2008 and hourly intraday price is sampled from 10: 30AM to 3: 30PM.

Over the sample period, several changes are made to the composition of the S&P 100 index. In order to ensure that our analysis is unbias and realistic, we keep a track record of all the additions and deletions to the index and ensure that we do not use any information for our stock selection that is not already public. The historical data of S&P 100 index components and associated addition and deletion is collected from the website of Standard & Poor's[2]. The list of additions and deletions over the *ex post* beta evaluation period is presented in Table 5.3.

5.5 Results

Figure 5.1 – Figure 5.3 show the history of quarterly returns for three momentum portfolios which start in different months in 1993, with comparison to that of the S&P 500 Index. These figures display a low correlation between the return on the index and the return on the momentum portfolio strategies. From early 1995 to the time prior to the bursting of the technology bubble in the early 2000s, all three momentum portfolios underperform the index. The index experienced a strong bullish trend with a cumulative return of over 300% at the peak. During the bursting of the technology bubble, generally all three momentum portfolios outperform the index, especially momentum portfolio 2. Furthermore, all three momentum portfolios have earned impressive positive returns

[2] www.standardandpoors.com

during the recent global financial crisis. These periods of crisis, illustrate the advantages of following momentum strategies and why they are so popular among equity market neutral hedge funds. However, although our three momentum portfolios follow similar P & L paths, they do end up with quite different final wealth. Therefore, including all three momentum portfolios, each starting in a different month, avoids data mining bias and offers an objective methodology to study the true performance accurately.

The momentum strategy is dollar neutral, which means the absolute value of long positions and short positions are equal, but not necessarily beta neutral. The S&P 500 E-Mini index futures are used to hedge residual beta exposure based on the four different beta forecasting techniques. Figure 5.4 - Figure 5.6 display the hedging dynamics for the three momentum portfolios. Although the general pattern of movement of the time series of hedging is similar for all four beta forecasts across all three momentum portfolios, the number of futures contracts needed to hedge residual beta exposure can vary substantially based on the beta forecast.

From the above discussion, it is evident that there is variability among the different beta forecasting techniques. To assess the accuracy of these forecasts, two portfolio weighting methods are used. Table 5.2 display the results when beginning-of-period equal stock weighting is used to calculate the *ex post* betas of the three momentum portfolios. If the beta forecast was perfectly accurate, the portfolio's *ex post* beta would be zero, which is shown in the last column as the filtered measured beta, being the HP100 filtered beta. The range of *ex post* beta is narrower for the annual realized beta and the AR(1) quarterly realized beta forecasts, compared to the Fama-MacBeth beta and quarterly realized beta forecasts. The annual realized beta forecast has the lowest Mean-Squared-Error (MSE) thus demonstrating the most portfolio beta-neutrality. The AR(1) quarterly realized beta forecast is the second best performer, followed by the quarterly realized beta forecast. The widely used Fama-MacBeth beta is the worst predictor, which has the highest variability. Typically, the Fama-MacBeth MSE is over four times that of the annual realized beta MSE. The annual realized beta forecast has a Mean-Absolute-Error (MAE) for each of our portfolios of 0.1170, 0.1201 and 0.1277, whereas the other beta forecasting methods typically have a MAE in excess of 0.2. Figure 5.7 - Figure 5.9 visually display these results by time series plots of *ex post* portfolio betas.

Table 5.3 presents the summary statistics on the *ex post* portfolio betas from monthly end-of-period weights for the three momentum portfolios. Similar results are found as in Table 5.2 from monthly beginning-of-period weights. Figure 5.10 - Figure 5.12 visually display these results by time series plots of *ex post* portfolio betas. Theses results indicate a robustness of our findings across the starting month of our momentum portfolios and also between beginning and end-of-month portfolio weights. In addition to reporting the MSE over our forecast evaluation periods, we also report the MSE separately for the first and second half of our forecast evaluation periods. Similar patterns in forecast errors between the different approaches exist over these different subsamples and thus we can conclude that our results can not be attributed to sampling variability in the data generating processes of stock returns. Demonstrating robustness of our results over subsamples of data in the current setting is adopted as an alternative to directly measuring statistical significance between approaches.

Furthermore, in Tables 5.2 and 5.3 we also calculate the summary statistics of forecast errors when we firstly assume that our forecast is the actual quarterly realized beta that occurred in the forecast period, denoted by measured beta. Similarly, we also calculate the summary statistics of forecast errors when we assume our forecast is the actual HP100 filtered quarterly realized beta that occurred in the forecast period. These results are reported to give ourselves a benchmark of summary statistics from a setting of forecasting with 100 per cent accuracy.

Finally, we conduct the robustness test by calculating realized beta from hourly returns over the recent period from Jan 2003 to Dec 2008. The proxy of the true realized beta is computed as the sum of the product of hourly stock returns and index returns divided by the sum of squared hourly index returns over the quarter.Tables 5.4 and 5.5 demonstrate the performnance of the four beta forecasting models with this high-frequency realized beta used as true beta, which is HF beta shown in the last column. Table 5.4 illustrates the results for three momentum portfolio when begining-of-period equal weighting is used. The range of *ex post* beta is narrower for the annualized realized beta and the AR(1) quarterly realized beta, which is consistent with the results displayed in Table 5.2. The annualized realized beta forecast has the lowest Mean-Squared-Error (MSE) and thus demonstrates the superior forecasting performance over the other three beta forecasting models. Consistent with results in Table 5.2, the AR(1) quaterly realized beta forecast comes the second, followed by the quaterly realized beta forecast. Not surprisingly, Fama-MacBeth beta is the worst forecaster. The MSE of the annualized realized beta forecast is around 0.30 in each of momentum portfolios, while Fama-MacBeth bea has a MSE around 0.50 in each portfolio. In other words, the reduction in MSE is about 40%.

Similar pattern is found in Mean-Absolute-Error (MAE). The Fama-MacBeth beta has a MAE in excess of that of the annualized realized beta forecast by 0.18 in each momentum portfolio and it implies about 40% - 50% reduction in MAE. The same pattern of forecast errors is presented in Table 5.3 where the end-of-period weighting is used. The annual realized beta forecast performs the best with an MSE of 0.08, 0.13 and 0.10 respectively in each portfolio. The AR(1) quaterly realized beta forecast and the quarterly realized beta forecast come in the second and third places. Fama-MacBeth beta has the largest MSE of 0.23, 0.27 and 0.25 in each portfolio, which is in sharp constrast with those of the annualized realized beta forecast. The order of MAE confirms the results in Table 5.3 again. The robustness of our results is further demonstrated in Table 5.4 and 5.5 where the hourly returns are used to compute the realized beta.

5.6 Conclusion

In this chapter, we build on recent literature that has highlighted some of the significant advantages of using higher frequency data to calculate and forecast the beta of stocks. We construct a momentum-based beta neutral equity portfolio using stocks comprising the S&P 100 index, and find that using daily data to evaluate realized betas allows us to better capture the true market exposure of the portfolio. In turn, this leads to market neutral portfolios that have substantially less unwanted systematic risk exposure. We believe that the inability of equity market neutral funds to exhibit market neutrality in their performance can, in large part, be attributed to the fact that they use out-dated and inaccurate beta estimation techniques when constructing their portfolios.

Table 5.1: Descriptive Statistics for returns of hedged momentum portfolios

		Fama-MacBeth beta	Quarterly realized beta	AR(1) Quarterly realized beta	Annual realized beta	Measured beta	Filtered measured beta
Port1	Mean	0.44%	-0.82%	0.39%	0.98%	1.71%	2.07%
	Std	13.47%	14.51%	13.82%	13.54%	12.33%	11.99%
	Min	-35.57%	-38.09%	-37.93%	-26.57%	-27.86%	-25.87%
	Max	32.02%	32.52%	32.86%	29.11%	27.29%	28.69%
	Skewness	-4.94%	-30.61%	-5.64%	-19.16%	-18.89%	-8.27%
	Kurtosis	391.87%	383.48%	382.41%	397.11%	374.24%	391.94%
Port2	Mean	0.85%	-0.56%	0.83%	1.35%	1.91%	3.64%
	Std	14.11%	14.94%	14.29%	12.61%	15.50%	11.56%
	Min	-39.84%	-41.26%	-39.89%	-16.66%	-45.03%	-17.87%
	Max	48.18%	47.87%	47.68%	21.55%	21.30%	21.08%
	Skewness	30.76%	3.60%	24.78%	22.70%	-150.88%	-24.85%
	Kurtosis	493.29%	475.31%	457.08%	146.64%	628.63%	196.32%
Port3	Mean	0.28%	-0.91%	0.26%	2.32%	0.74%	2.74%
	Std	16.28%	16.46%	16.23%	8.44%	14.26%	8.91%
	Min	-40.75%	-42.79%	-45.43%	-15.20%	-45.39%	-15.32%
	Max	56.35%	56.63%	56.82%	14.70%	20.32%	19.87%
	Skewness	22.80%	12.78%	9.83%	-36.27%	-188.31%	-8.89%
	Kurtosis	553.63%	552.29%	588.02%	295.90%	821.02%	266.66%

This table presents the descriptive statistics for returns of our 3 hedged overlapping portfolios. We show the mean, standard deviation, minimum, maximum, skew and kurtosis for portfolio built on the 4 beta forecasting models.

Table 5.2: Ex-post portfolio analysis using equal-weights

		Fama-MacBeth beta	Quarterly realized beta	AR(1) Quarterly realized beta	Annual realized beta	Measured beta	Filtered measured beta
	mean	0.0510	-0.0161	0.0630	0.0606	-0.0233	0.0000
	std	0.3615	0.3190	0.2136	0.1398	0.3026	0.0000
portfolio 1	min	-0.8532	-0.8629	-0.5087	-0.2341	-0.6336	0.0000
	max	0.9512	1.1213	0.5953	0.4663	0.9856	0.0000
	MSE	0.1312	0.1004	0.0489	0.0229	0.0907	0.0000
	MAE	0.2837	0.2443	0.2494	0.1170	0.2362	0.0000
	MSE(1st Sub-period)	0.1355	0.0768	0.0321	0.0162	0.0605	0.0000
	MSE(2nd Sub-period)	0.1269	0.1240	0.0657	0.0296	0.1209	0.0000
	mean	0.0745	-0.0026	0.0677	0.0655	-0.0228	0.0000
	std	0.3679	0.2884	0.2179	0.1459	0.2684	0.0000
portfolio 2	min	-0.7272	-0.4880	-0.5582	-0.2461	-0.4673	0.0000
	max	0.9940	0.9886	0.5701	0.5250	1.1477	0.0000
	MSE	0.1387	0.0819	0.0513	0.0252	0.0714	0.0000
	MAE	0.2876	0.2297	0.1756	0.1201	0.1971	0.0000
	MSE(1st Sub-period)	0.1484	0.0530	0.0527	0.0204	0.0430	0.0000
	MSE(2nd Sub-period)	0.1287	0.1117	0.0498	0.0302	0.1007	0.0000
	mean	0.0745	-0.0053	0.0698	0.0742	-0.0251	0.0000
	std	0.3477	0.2620	0.2255	0.1487	0.2858	0.0000
portfolio 3	min	-0.8138	-0.8013	-0.4418	-0.2682	-0.6945	0.0000
	max	0.8312	0.6922	0.5917	0.5024	1.2082	0.0000
	MSE	0.1246	0.0676	0 .0549	0.0273	0.0810	0.0000
	MAE	0.2896	0.1918	0.1856	0.1277	0.0962	0.0000
	MSE(1st Sub-period)	0.1166	0.0592	0.0433	0.0279	0.0589	0.0000
	MSE(2nd Sub-period)	0.1328	0.0762	0.0670	0.0267	0.1038	0.0000

This table presents the descriptive statistics for the ex-post beta exposure of our 3 overlapping portfolios. We show the mean, standard deviation, minimum, maximum, mean-squared-error and mean-absolute-error for the 4 beta models forecast error, as well as for the measured beta and filtered measured beta assuming the stocks have an equal-weight in the portfolio. The quarterly realized beta is calculated from daily returns over the period and smoothed by HP100 filter. The forecasting evaluation period is from January 2, 1993 to December 31, 2008.

Table 5.3: Ex-post analysis using end-of-period weights

		Fama-MacBeth beta	Quarterly realized beta	AR(1) Quarterly realized beta	Annual realized beta	Measured beta	Filtered measured beta
portfolio 1	mean	0.0607	-0.0065	0.0726	0.0703	-0.0136	0.0097
	std	0.3538	0.3204	0.1951	0.1377	0.3236	0.0525
	min	-0.8517	-0.8968	-0.3968	-0.2257	-0.6675	-0.1448
	max	0.9377	0.9945	0.5820	0.4640	1.1615	0.1760
	MSE	0.1269	0.1011	0.0428	0.0236	0.1033	0.0028
	MAE	0.2841	0.2502	0.1587	0.1187	0.2497	0.0367
	MSE(1st Sub-period)	0.1369	0.0824	0.0335	0.0186	0.0668	0.0012
	MSE(2nd Sub-period)	0.1169	0.1198	0.0520	0.0286	0.1398	0.0044
portfolio 2	mean	0.0947	0.0177	0.0880	0.0858	-0.0025	0.0203
	std	0.3586	0.3109	0.2055	0.1504	0.2910	0.0541
	min	-0.7048	-0.4833	-0.3820	-0.2011	-0.4379	-0.0567
	max	1.0005	1.1164	0.5766	0.5467	1.2755	0.2886
	MSE	0.1355	0.0954	0.0493	0.0296	0.0833	0.0033
	MAE	0.2865	0.2439	0.1774	0.1316	0.1032	0.0347
	MSE(1st Sub-period)	0.1533	0.0617	0.0583	0.0254	0.0465	0.0014
	MSE(2nd Sub-period)	0.1171	0.1302	0.0401	0.0340	0.1214	0.0053
portfolio 3	mean	0.0380	-0.0497	0.0395	0.0401	-0.0382	-0.0155
	std	0.4058	0.3657	0.2914	0.2883	0.4050	0.2413
	min	-0.8602	-0.7900	-0.7473	-0.7964	-0.9953	-1.0101
	max	1.1811	0.8286	0.6603	0.7260	1.1543	0.5548
	MSE	0.1635	0.1341	0.0851	0.0834	0.1629	0.0575
	MAE	0.3136	0.2896	0.2277	0.2229	0.3225	0.1633
	MSE(1st Sub-period)	0.1526	0.1277	0.0739	0.0791	0.1181	0.00639
	MSE(2nd Sub-period)	0.1747	0.1406	0.0967	0.0878	0.2091	0.0510

This table presents the descriptive statistics for the ex-post beta exposure of our 3 overlapping portfolios. We show the mean, standard deviation, minimum, maximum, mean-squared-error and mean-absolute-error for the 4 beta models forecast error, as well as for the measured beta and filtered measured beta using the actual weights of the stocks in the portfolio at the end of the holding period. The quarterly realized beta is calculated from daily returns over the period and smoothed by HP100 filter. The forecasting evaluation period is from January 2, 1993 to December 31, 2008.

Table 5.4: Ex-post portfolio analysis using equal-weights with HF data

		Fama-MacBeth beta	Quarterly realized beta	AR(1) Quarterly realized beta	Annual realized beta	HF beta
	mean	-0.0759	0.1055	0.0539	0.0754	0.0000
	std	0.4871	0.4105	0.3050	0.2781	0.0000
portfolio 1	min	-0.9022	-0.3331	-0.4872	-0.4241	0.0000
	max	0.8697	1.7391	0.6215	0.6978	0.0000
	MSE	0.2331	0.1726	0.0920	0.0798	0.0000
	MAE	0.4055	0.2368	0.2437	0.2206	0.0000
	mean	-0.0423	0.0701	0.0612	0.0805	0.0000
	std	0.5283	0.5390	0.3821	0.3533	0.0000
portfolio 3	min	-1.0177	-0.5763	-0.9029	-0.6173	0.0000
	max	0.8389	2.0519	0.8127	0.9392	0.0000
	MSE	0.2688	0.2828	0.1434	0.1259	0.0000
	MAE	0.4501	0.3373	0.2793	0.2692	0.0000
	mean	-0.0405	0.0609	0.0398	0.0820	0.0000
	std	0.5056	0.4185	0.3342	0.3203	0.0000
portfolio 3	min	-1.2190	-0.6027	-0.7420	-0.7048	0.0000
	max	0.7251	1.5777	0.6847	0.7455	0.0000
	MSE	0.2462	0.1712	0.1084	0.1048	0.0000
	MAE	0.3803	0.2423	0.2413	0.2305	0.0000

This table presents the descriptive statistics for the ex-post beta exposure of our 3 overlapping portfolios. We show the mean, standard deviation, minimum, maximum, mean-squared-error and mean-absolute-error for the 4 beta models forecast error, as well as for filtered measured beta using the actual weights of the stocks have an equal-weight in the portfolio. The quarterly realized beta is calculated from hourly returns over the period. The forecasting evaluation period is from January 2, 2003 to December 31, 2008.

Table 5.5: Ex-post analysis using end-of-period weights

		Fama-MacBeth beta	Quarterly realized beta	AR(1) Quarterly realized beta	Annual realized beta	HF beta
	mean	-0.0759	0.1140	0.0539	0.0754	0.0103
	std	0.4871	0.4070	0.3050	0.2781	0.0435
portfolio 1	min	-0.9022	-0.3260	-0.4872	-0.4241	-0.1005
	max	0.8697	1.7277	0.6215	0.6978	0.1157
	MSE	0.2331	0.1718	0.0920	0.0798	0.0019
	MAE	0.4055	0.2447	0.2437	0.2206	0.1005
	mean	-0.0423	0.0869	0.0612	0.0805	0.0180
	std	0.5283	0.4996	0.3821	0.3533	0.0902
portfolio 2	min	-1.0177	-0.6185	-0.9029	-0.6173	-0.2091
	max	0.8389	1.8428	0.8127	0.9392	0.3325
	MSE	0.2688	0.2463	0.1434	0.1259	0.0081
	MAE	0.4501	0.3225	0.2793	0.2692	0.0478
	mean	-0.0405	0.0465	0.0398	0.0820	0.0335
	std	0.5056	0.4558	0.3342	0.3203	0.3096
portfolio 3	min	-1.2190	-0.8747	-0.7420	-0.7048	-0.5603
	max	0.7251	0.7638	0.6847	0.7455	0.7720
	MSE	0.2462	0.2009	0.1084	0.1048	0.0928
	MAE	0.3803	0.3464	0.2413	0.2305	0.0575

This table presents the descriptive statistics for the ex-post beta exposure of our 3 overlapping portfolios. We show the mean, standard deviation, minimum, maximum, mean-squared-error and mean-absolute-error for the 4 beta models forecast error, as well as for the measured beta and filtered measured beta using the actual weights of the stocks in the portfolio at the end of the holding period. The quarterly realized beta is calculated from hourly returns over the period. The forecasting evaluation period is from January 2, 2003 to December 31, 2008.

Table 5.6: S&P 100 Index addition and deletion from 1993 to 2008

Date	Addition-Company	Addition-Ticker	Deletion-Company	Deletion-Ticker
05-Mar-93	Intel Corp.	INTC	Humana Inc.	HUM
22-Nov-93	May Dept. Stores	MAY	Paramount Communications	PCI
17-Mar-94	Chrysler Corp.	C	Litton Industries	LIT
12-Jul-94	First Fidelity Bancorp	FFWV	UAL Corp.	UAL
30-Jun-95	Harrah's Entertainment	HET	Promus Companies	PRI
28-Jul-95	Nynex	NYN	Skyline Corp.	SKY
02-Nov-95	Pharmacia & Upjohn, Inc.	PNU	Upjohn Co.	UPJ
30-Nov-95	First Chicago NBD Corp.	FCN	First Chicago Corp.	FCN
19-Dec-95	ITT Hartford Group, Inc.	HIG	ITT Corp.	ITT
29-Dec-95	Columbia/HCA Healthcare Corp.	HCA	First Fidelity Bancorp	FFWV
09-Feb-96	Oracle Systems	ORCL	Capital Cities/ABC	CCB
29-Mar-96	Cisco Systems	CSCO	First Interstate Bancorp	I
15-Aug-96	Allegheny Teledyne Inc.	ATI	Teledyne Inc.	TDY

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Table 5.6 – Continued

Date	Addition-Company	Additions-Ticker	Deletions-Company	Deletions-Ticker
01-Jul-97	NationsBank	NB	Great Western Financial	GWF
15-Aug-97	Microsoft	MSFT	Nynex	NYN
27-Jan-98	FDX Holding Corp.	FDX	Federal Express	FDX
11-Jun-98	Procter & Gamble	PG	Digital Equipment	DEC
15-Sep-98	Bank One Corp.	ONE	MCI Communications	MCIC
30-Sep-98	Campbell Soup	CPB	BankAmerica Corp.	BAC
30-Sep-98	BankAmerica Corp. (New)	BAC	NationsBank	NB
01-Oct-98	U.S. Bancorp	USB	First Chicago NBD Corp.	FCN
07-Oct-98	Citigroup Inc.	CCI	Citicorp	CCI
12-Nov-98	CBS Corporation	CBS	Chrysler Corporation	C
31-Dec-98	Wells Fargo & Co.	WFC	Amoco Corp.	AN
01-Apr-99	Lucent Technologies	LU	AMP Inc.	AMP
12-Oct-99	Home Depot	HD	Ameritech	AIT
05-Nov-99	Amgen	AMGN	Harris Corp.	HRS
29-Nov-99	Allegheny Technologies Inc.	ATI	Allegheny Teledyne Inc.	ATI

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Table 5.6 – Continued

Date	Addition-Company	Additions-Ticker	Deletions-Company	Deletions-Ticker
30-Nov-99	Sara Lee Corp.	SLE	Mobil Corp.	MOB
01-Dec-99	Honeywell Int'l Inc.	HON	Honeywell	HON
29-Aug-01	SBC Communoications	SBC	American General	AGC
09-Oct-01	Phillip Morris	PM	Global Crossing	GX
29-Nov-01	Anheuser-Busch	BUD	Enron Corp.	ENE
13-Dec-01	Medtronic Inc.	MDT	Ralston-Ralston Purina	RAL
19-Jul-02	Goldman Sachs Group	GS	Nortel Networks Corp. Hldg. Co.	NT
15-Apr-03	Allstate Corp.	ALL	Pharmacia Corp.	PHA
31-Mar-00	Morgan Stanley Dean Witter	MWD	Monsanto Company	MTC
31-Mar-00	Pharmacia Corp.	PHA	Pharmacia & Upjohn Inc.	PNU
17-Apr-00	EMC Corp.	EMC	Atlantic Richfield	ARC
	Viacom Inc.	VIA	CBS Corp.	CBS
16-Jun-00	America Online	AOL	Champion International	CHA
17-Oct-00	Tyco International	TYC	Mallinckrodt Inc.	MKG
20-Oct-00	Exelon Corp	EXC	Unicom Corp	UCM

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Table 5.6 – Continued

Date	Addition-Company	Additions-Ticker	Deletions-Company	Deletions-Ticker
08-Dec-00	Enron Corp.	ENE	Bethlehem Steel	BS
11-Dec-00	Gillette Co.	G	Polaroid Corp.	PRD
15-Dec-00	Chase Manhattan Corp.	CMB	Ceridian Corp.	CEN
15-Dec-00	Pfizer Inc.	PFE	K Mart	KM
15-Dec-00	Lehman Bros. Hldgs	LEH	Brunswick Corp.	BC
15-Dec-00	Clear Channel Communications	CCU	Tektronix Inc.	TEK
15-Dec-00	AES Corp.	AES	Homestake Mining	HM
15-Dec-00	NEXTEL Communications	NXTL	Massey Energy Co.	MEE
15-Dec-00	MedImmune Inc.	MEDI	Occidental Petroleum	OXY
15-Dec-00	Global Crossing Ltd.	GX	International Flav/Frag.	IFF
29-Jan-01	El Paso Energy	EPG	Coastal Corp.	CGP
26-Feb-01	Firstar Corp.	FSR	U.S. Bancorp	USB
29-Aug-01	SBC Communications	SBC	American General	AGC
09-Oct-01	Philip Morris	MO	Global Crossing Ltd.	GX
29-Nov-01	Anheuser-Busch	BUD	Enron Corp.	ENE

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Table 5.6 – Continued

Date	Addition-Company	Additions-Ticker	Deletions-Company	Deletions-Ticker
12-Dec-01			Ralston-Ralston Purina Gp	RAL
13-Dec-01	Medtronic Inc.	MDT		
19-Jul-02	Goldman Sachs Group	GS	Nortel Networks	NT
15-Apr-03	Allstate Corp.	ALL	Pharmacia Corp.	PHA
30-Jun-04	Dell Inc	DELL	Bank One Corp.	ONE
24-Mar-05	Comcast Corp.	CMCSA	Sears Roebuck & Co.	S
21-Jul-05	Target Corp.	TGT	Toys "R" US	TOY
12-Aug-05	Sprint Corp.	FON	Nextel Communications	NXTL
18-Aug-05	Caterpiller Inc.	CAT	Delta Air Lines	DAL
29-Aug-05	Abbott Laboratories	ABT	May Dept. Stores	MAY
30-Sep-05			Gillette Co.	G
03-Oct-05	Chevron Corp.	CVX		
18-Nov-05	United Parcel Services	UPS		
30-Dec-05	CBS Corp.	CBS	Viacom Inc. (Old)	VIA
29-Sep-06	Wachovia Corp.	WB	Unisys Corp.	UIS

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Table 5.6 – Continued

Date	Addition-Company	Additions-Ticker	Deletions-Company	Deletions-Ticker
03-Nov-06	Regions Financial Corp.	RF	RadioShack	RSH
17-Nov-06	Google Inc.	GOOG	HCA Inc.	HCA
30-Nov-06	Capital One Fincl. Corp.	COF	OfficeMax Inc.	OMX
30-Nov-06	Conoco Phillips	COP	Lucent Technologies	LU
22-Mar-07	CVS/Caremark Corp.	CVS	Black & Decker	BDK
30-Mar-07	Kraft Foods Inc. 'A'	KFT	Eastman Kodak	EK
	Apple Inc.	AAPL	MedImmune Inc.	MEDI
29-Jun-07	Tyco Int'l (new)	TYC	Tyco Int'l (old)	TYC
29-Jun-07	Bank of New York Mellon Corp. (new)	BK	Computer Sciences	CSC
29-Jun-07	Covidien Ltd.	COV	National Semiconductor Corp.	NSM
24-Oct-07	NYSE Euronext Inc.	NYX	Limited Brands	LTD
28-Jan-08	UnitedHealth Group	UNH	Harrarh's Entertainment	HET
31-Mar-08	Philip Morris International	PM	Allegheny Technologies Inc.	ATI
22-Apr-08	National Oilwell Varco Inc.	NOV	Rockwell Automation Inc.	ROK
17-Jul-08	MasterCard Inc.	MA	General Motors Corp.	GM

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Table 5.6 – Continued

Date	Addition-Company	Additions-Ticker	Deletions-Company	Deletions-Ticker
30-Jul-08	Qualcomm Inc.	QCOM	Clear Channel Communications Inc.	CCU
16-Sep-08			Lehman Brothers Holdings Inc.	LEH
19-Sep-08	Occidental Petroleum Corp.	OXY		
18-Nov-08	Wyeth	WYE	Anheuser-Busch	BUD
19-Dec-08	Nike Inc.	NKE	CIGNA Corp.	CI
19-Dec-08	Walgreen Co.	WAG	Hartford Financial Services Group Inc.	HIG
19-Dec-08	Gilead Sciences Inc.	GILD	International Paper Co.	IP
19-Dec-08	Lowe's Companies Inc.	LOW	CBS Corp.	CBS
19-Dec-08	Devon Energy Corp.	DVN	El Paso Corp.	EP
19-Dec-08	Lockheed Martin Corp.	LMT	The AES Corp.	AES
19-Dec-08	Schering-Plough Corp.	SGP	American International Group Inc.	AIG
31-Dec-08	Amazon.com Inc.	AMZN	Merrill Lynch & Co. Inc.	MER
31-Dec-08	Costco Wholesale Corp.	COST	Wachovia Corp.	WB

1.although CVX was added on , to make sure there are always 100 stocks in the index, we assume we already know CVX would be added on .

2.AT&T was merged with SBC Communication in Nov 2005 and both were in S & P 100 Index. Therefore, only one addition while no deletion was made.

3.The addition & deletion information over the period from 2000 to 2008 is downloaded from www.standardandpoor.com. The data prior to 200 was purchased from Standard and Poor's.

Figure 5.1: Cumulative return for momentum portfolio 1 and S&P 500 market index

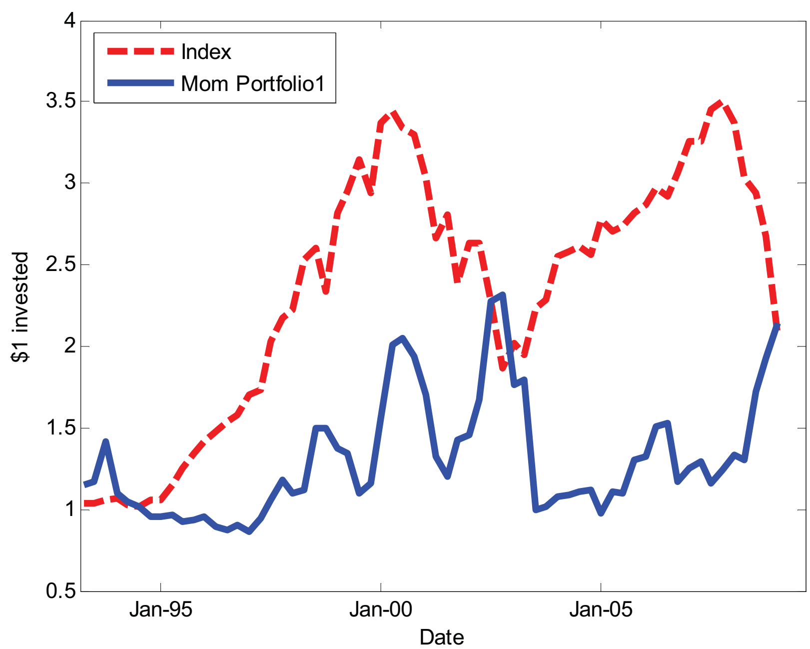


Figure 5.2: Cumulative return for momentum portfolio 2 and S&P 500 market index

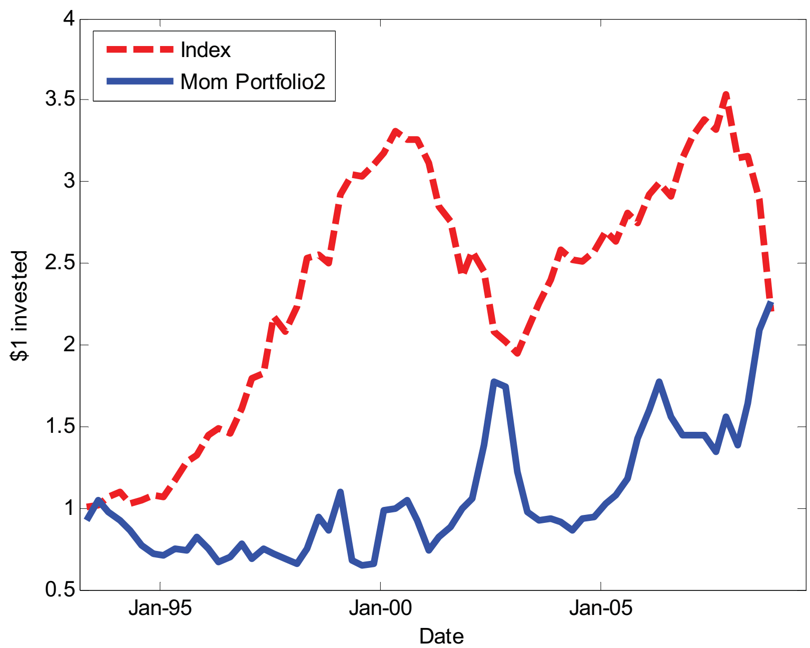


Figure 5.3: Cumulative return for momentum portfolio 3 and S&P 500 market index

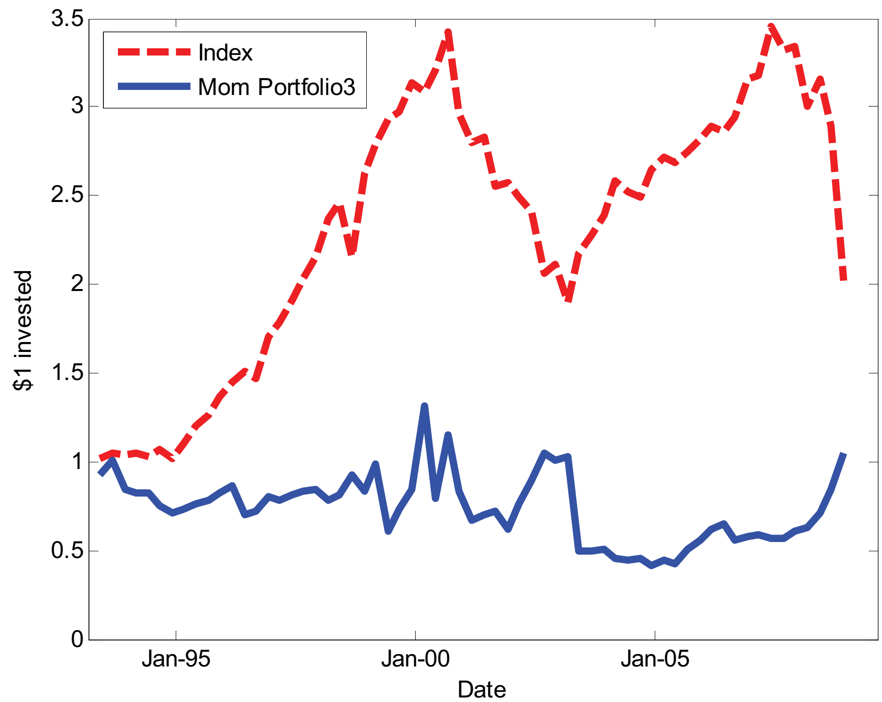


Figure 5.4: Dynamics of hedge for momentum portfolio 1

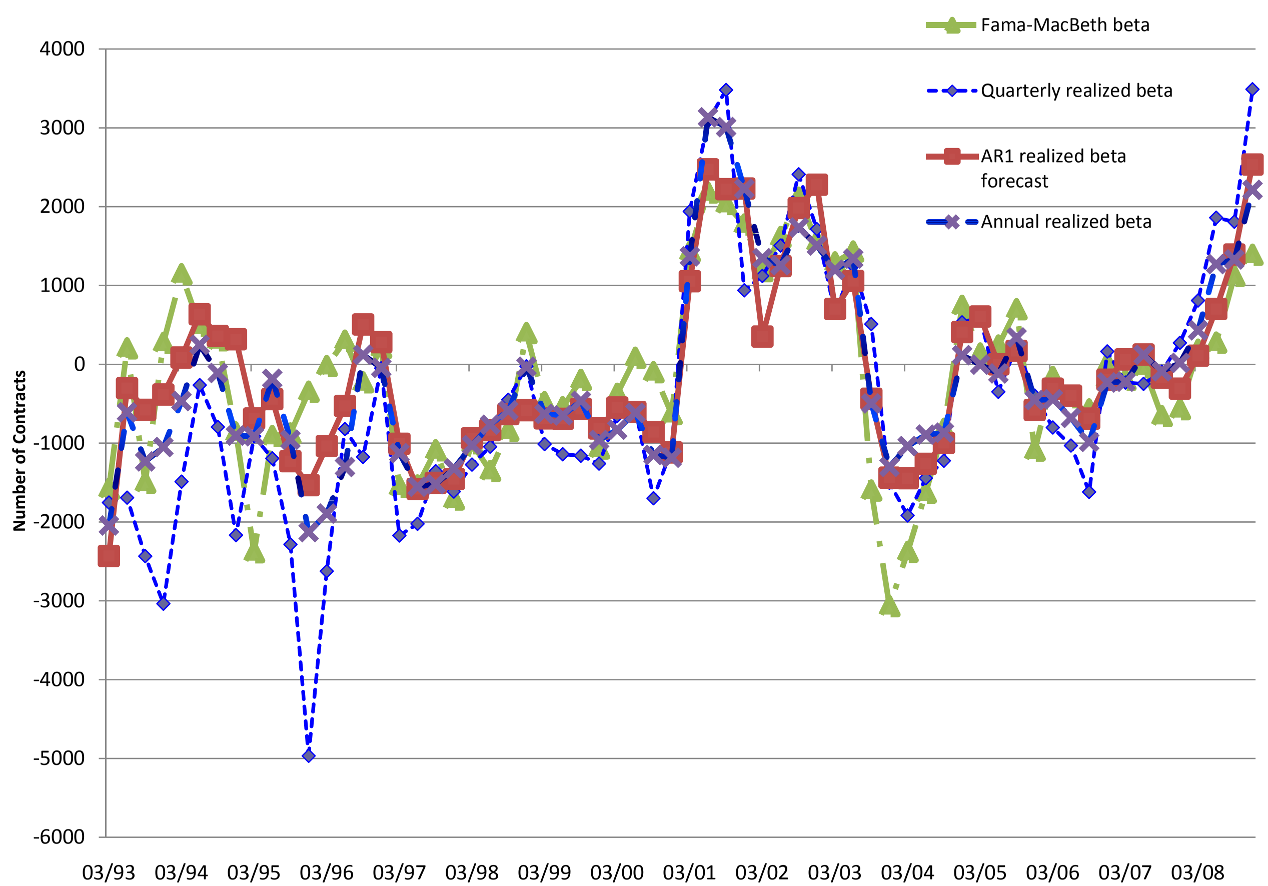
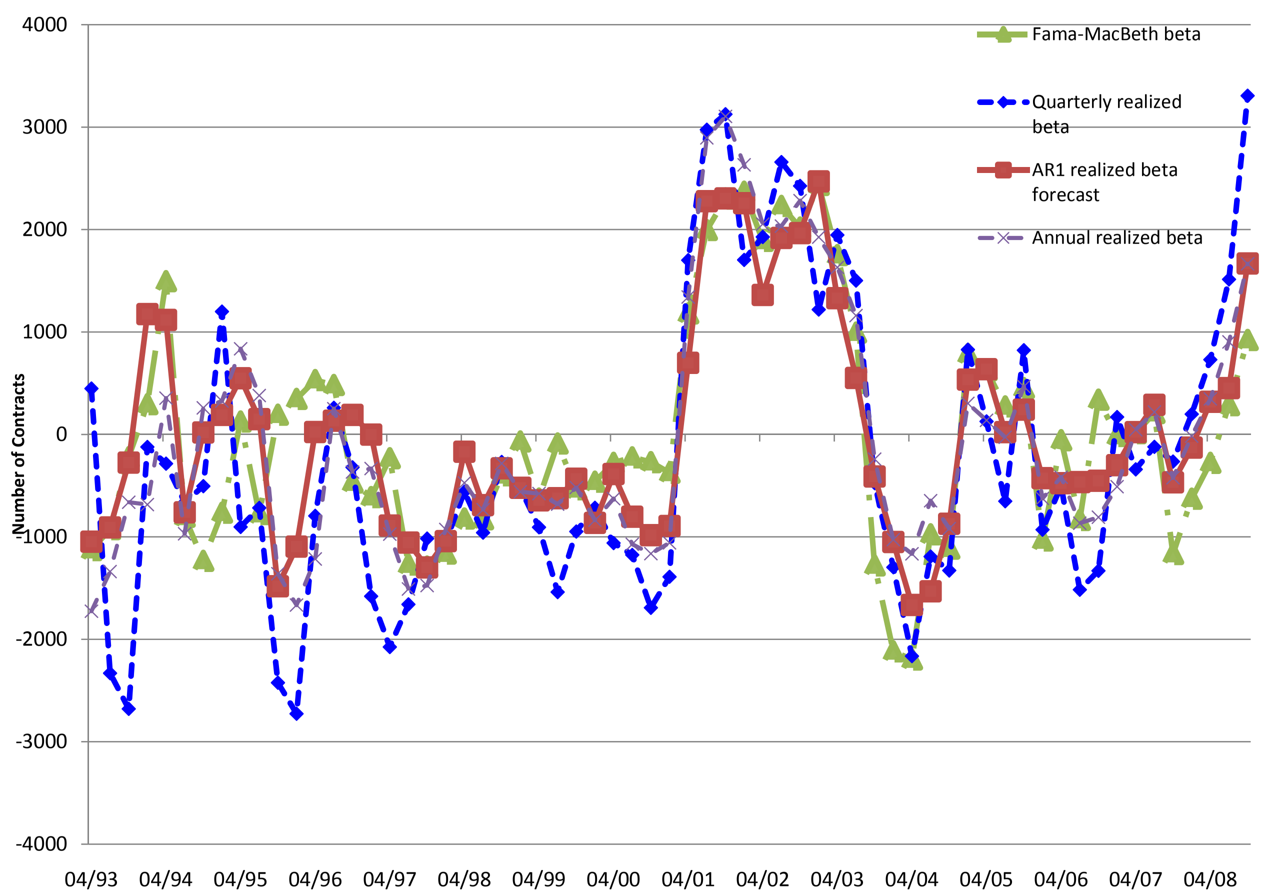


Figure 5.5: Dynamics of hedge for momentum portfolio 2



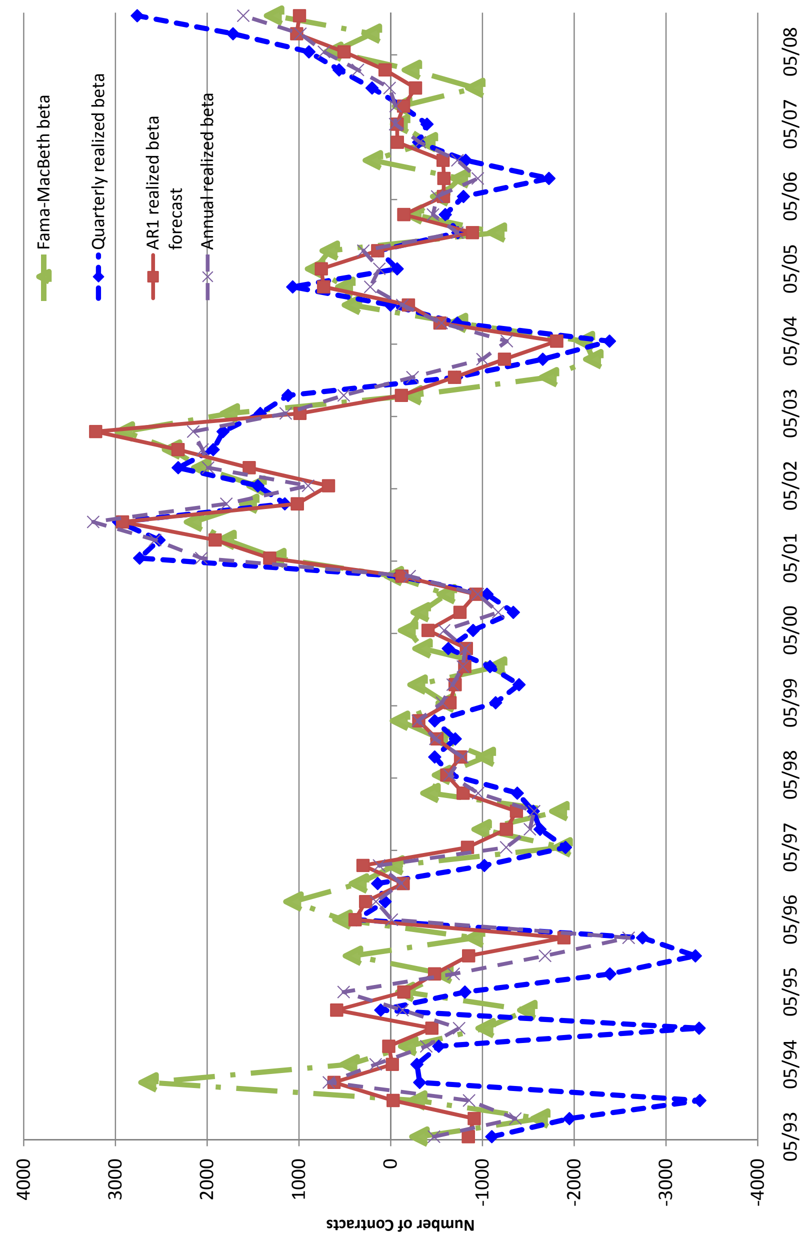


Figure 5.6: Dynamics of hedge for momentum portfolio 3

Figure 5.7: Ex post beta exposure for momentum portfolio 1 using equal weights

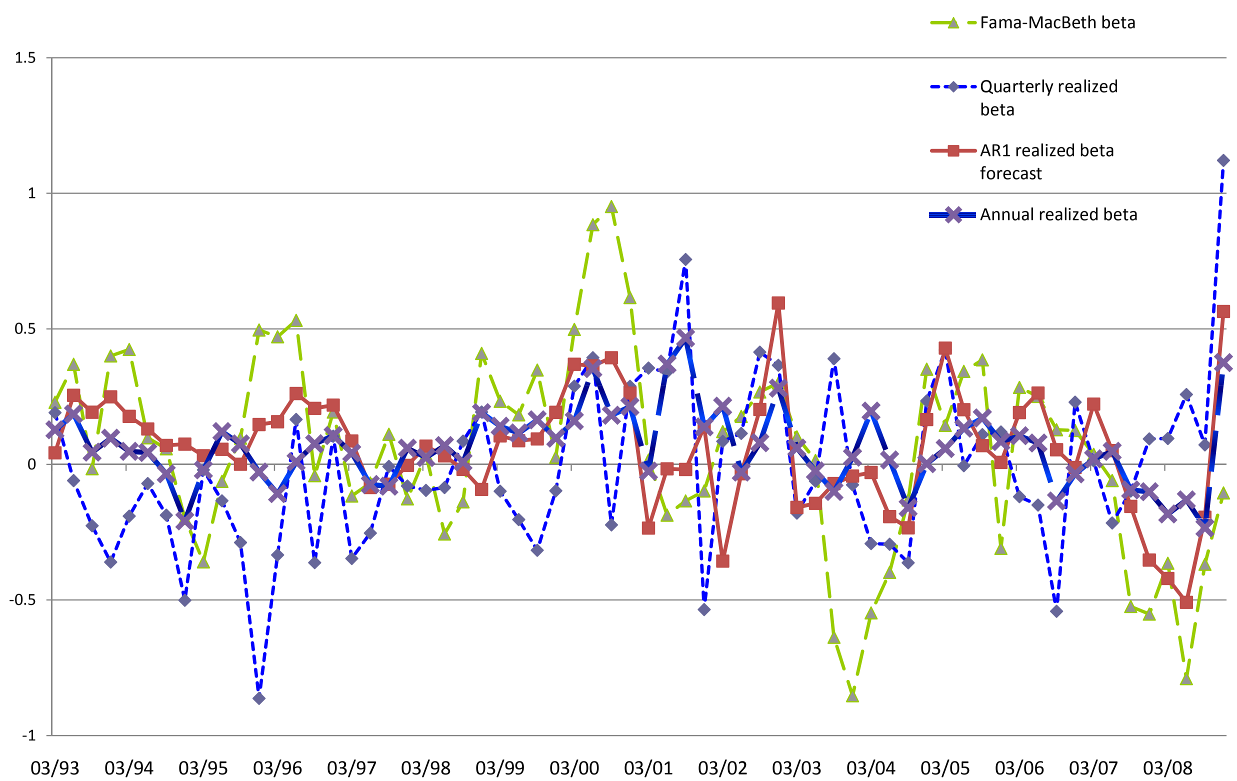


Figure 5.8: Ex post beta exposure for momentum portfolio 2 using equal weights

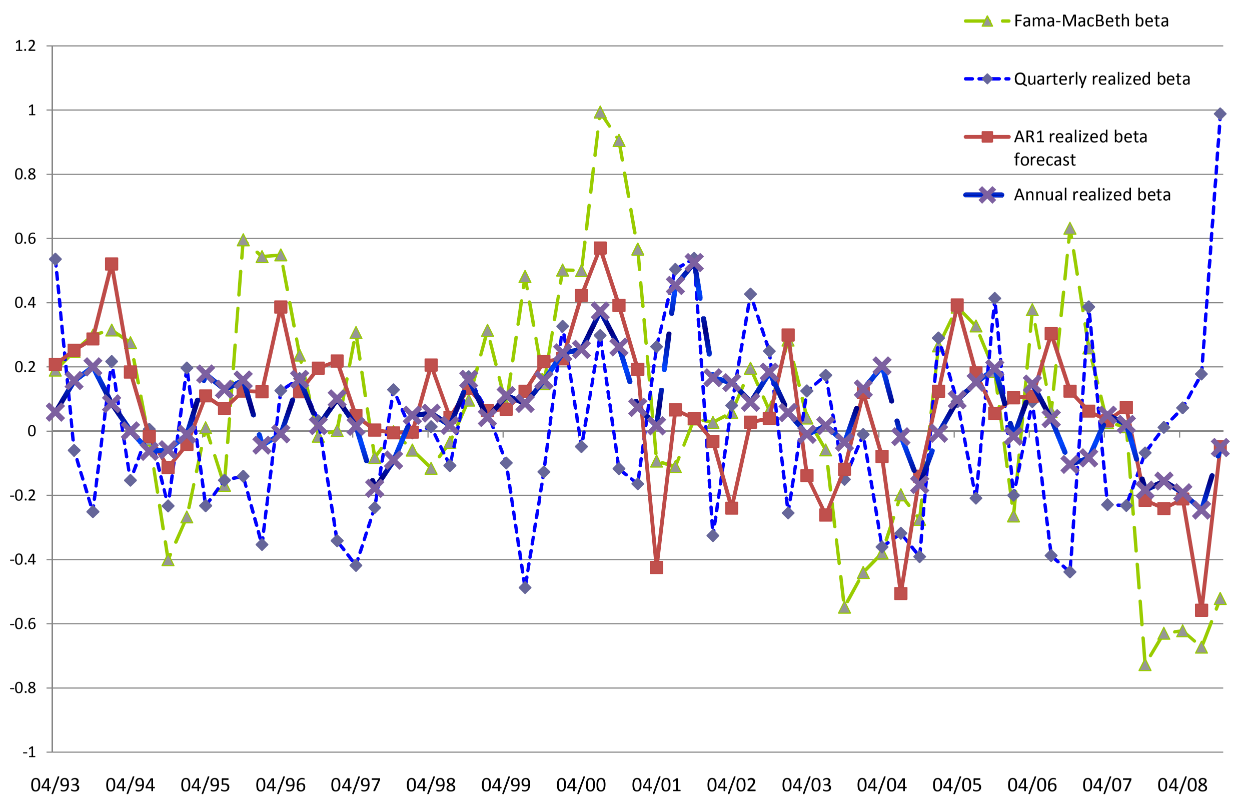


Figure 5.9: Ex post beta exposure for momentum portfolio 3 using equal weights

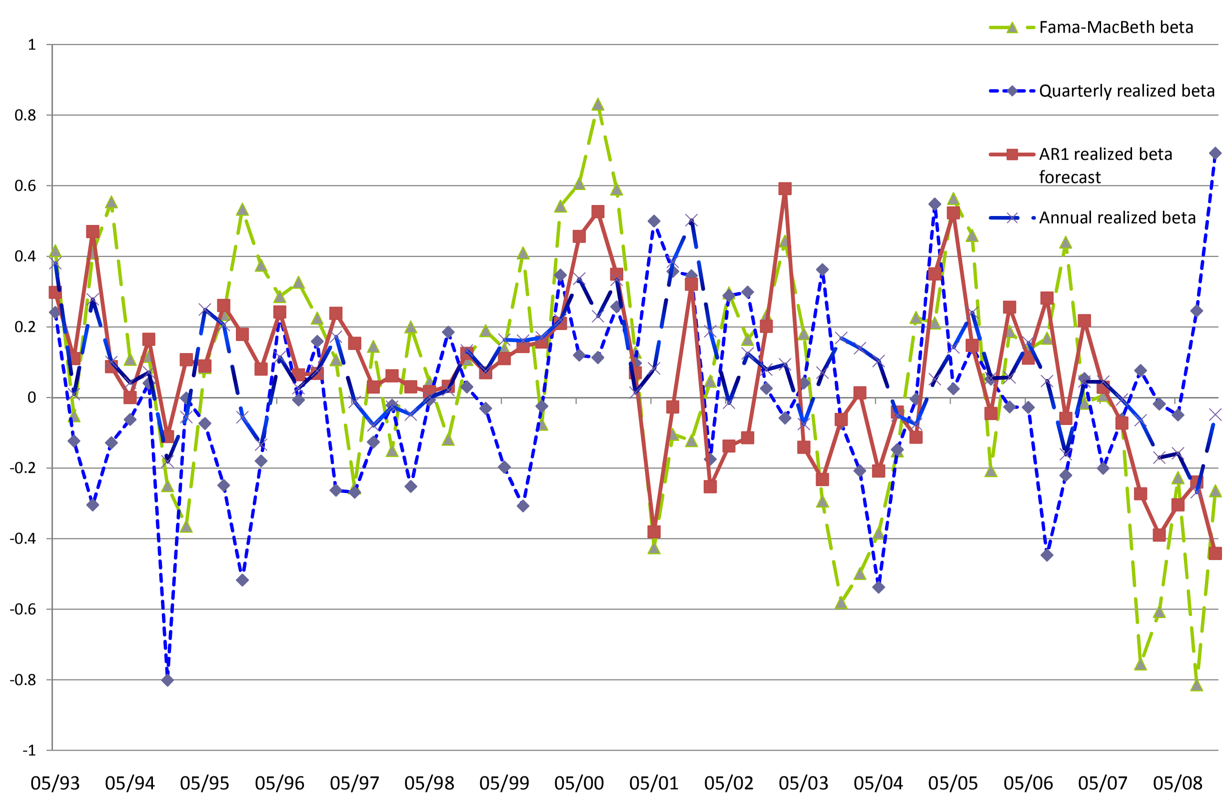


Figure 5.10: Ex-post beta exposure for momentum portfolio 1 using end-of-period weights

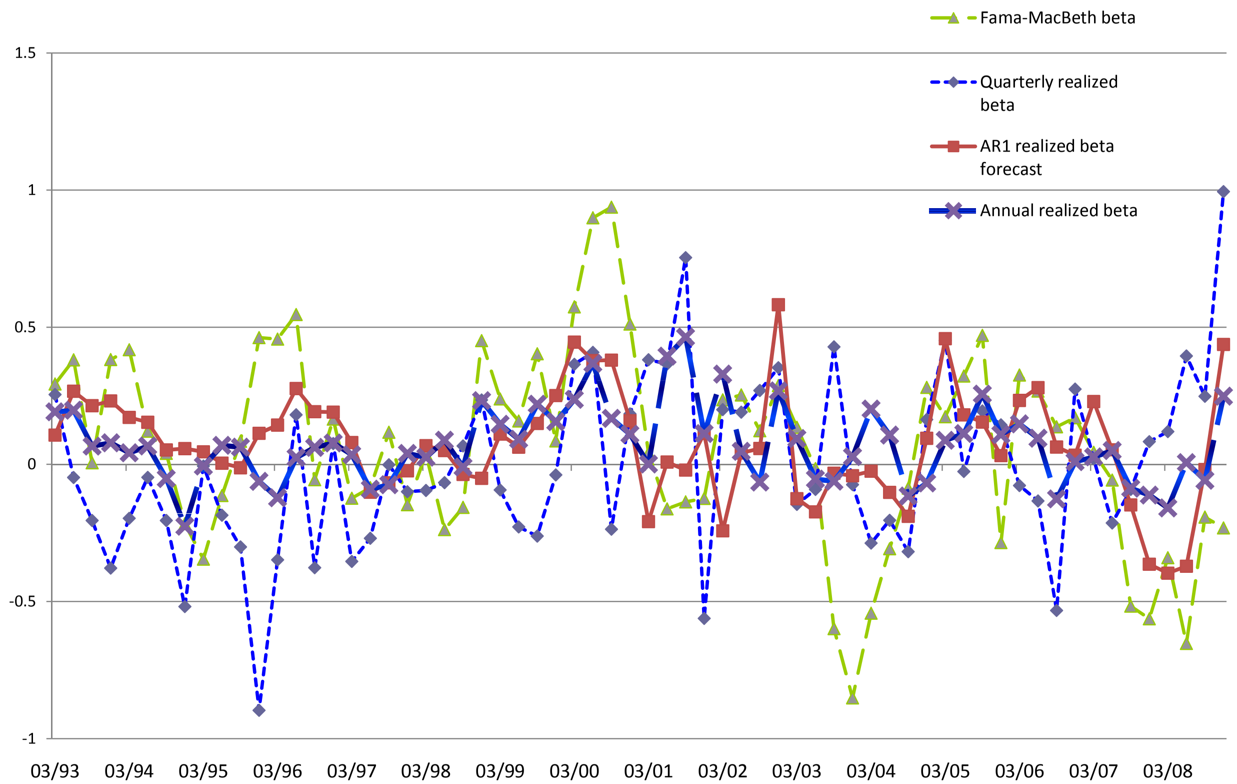


Figure 5.11: Ex-post beta exposure for momentum portfolio 2 using end-of-period weights

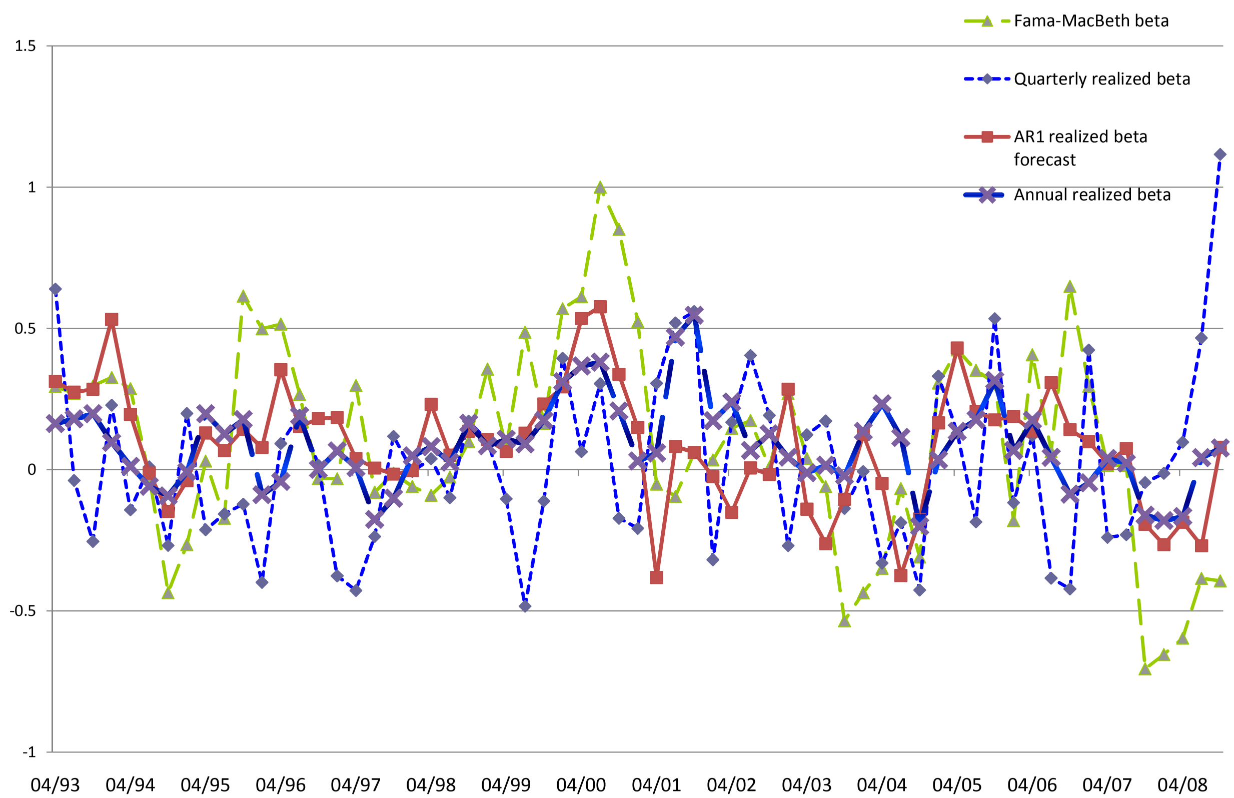
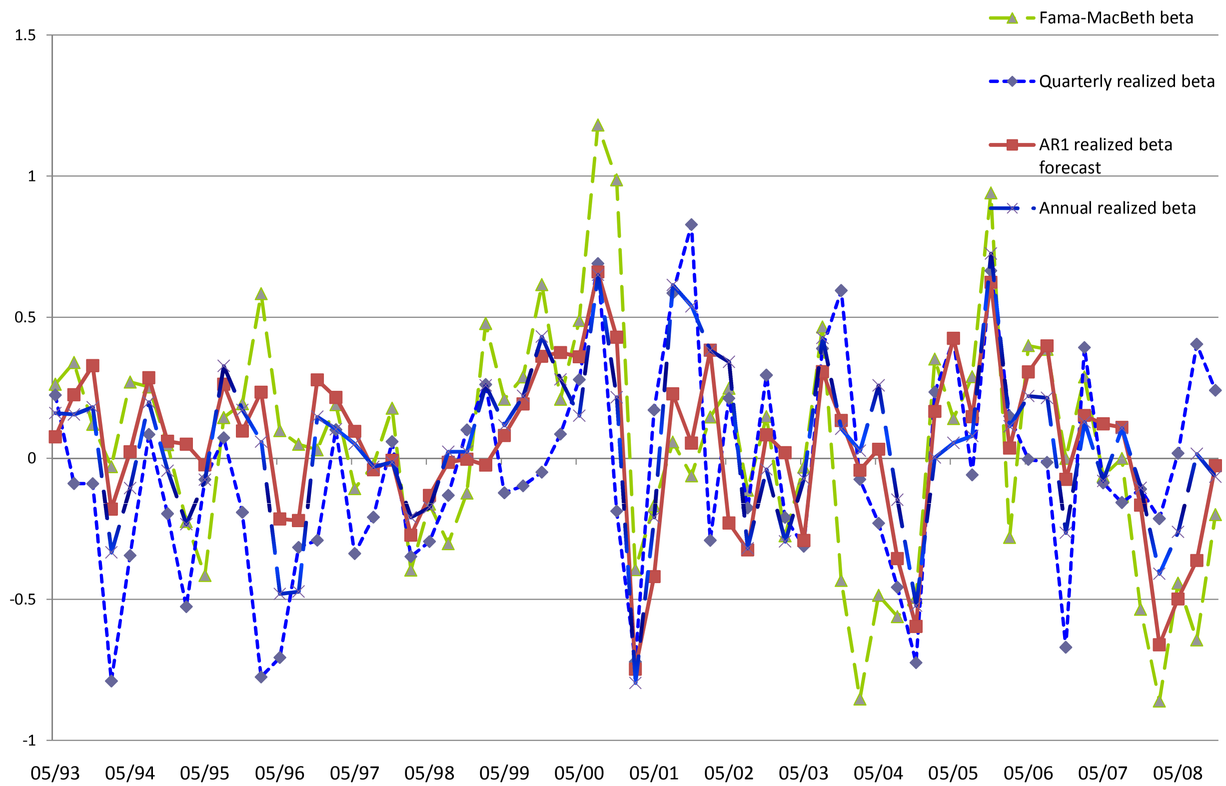


Figure 5.12: Ex-post beta exposure for momentum portfolio 3 using end-of-period weights



Chapter 6

Concluding Remarks

This thesis contributes to the growing literature on realized volatility and realized beta. In chapter two, structural stability is tested on models of realized volatility with a sub-sampling method. Andrews (2003) test is used to detect breaks on foreign exchange realized volatility time series. A high correlation is found between unstable observations in realized volatility and realized bi-power variation, which might suggest that the breaks are common to both measures of volatility. It's also found a very small portion of observations are identified as breaks. The method of removing structurally unstable data of a short duration has a negligible impact on the accuracy of conditional mean forecasts of volatility. On the other hand, it does improve the forecast density of volatility. In addition, the forecasting performance on structurally stable data improves dramatically. The result is beneficiary to risk managers, who is interested in the risk distribution.

There are numerous papers on volatility forecasting at the frequencies of daily, weekly and monthly. Chapter three complements the existing literature on volatility forecasting by investigating stock return volatility forecasting techniques at the quarterly frequency. Following Martens et al. (2008), the quarterly realized volatility, which is used as the proxy for the unobserved and underlying volatility, is contructed from 30-minute and overnight returns due to the fact that overnight volatility is an important part of stock return volatility. The forecasting variables are the past realized volatility constructed from daily return. It is found that an autoregressive model with one lag of quarterly realized volatility produces the most accurate forecasts, and dominates other approaches, including the recently proposed MIDAS approach. The investor can still obtain an accurate quarterly volatility with this simple model with free daily return time series without purchasing and processing expensive high-frequency data.

Chapter four extends Chen and Reeves (2009) by examining the measurement of security beta's constructed from daily returns at the quarterly frequency and smoothed with the HP filter. The results are consistent with Chen and Reeves (2009). The measurement error is reduced by approximately 50% compared to the quarterly realized beta from daily returns, and by two-thirds relative to the Fama-MacBeth beta. Therefore, it's a useful technique to investors who do not have access to high-quality high-frequency price data due to high cost and complexity of processing high-frequency data. The result is also used to construct the proxy for underlying quarterly realized beta time series in chapter five.

Chapter five evaluates realized beta forecasting techniques in order to achieve market neutrality. The recent literature has demonstrated superior advantages of higher frequency data to access stock betas. The evaluation is built on the momentum-based beta neutral equity portfolio with stock components in the S&P 100 index. The annualized realized beta forecast, which is constructed from high-frequency data, demonstrates the closest to beta neutrality in each of our momentum portfolios. The commonly used Fama-MacBeth beta has the worst forecasting performance. Furthermore, our result is robust when hourly returns over the recent years are used to calculate the proxy for the true realized beta. The results highlight the benefit of utilizing daily data for stock beta construction. The high correlation of market neutral funds and market stock index exhibited during financial crisis, in large part, could be attributed to the fact that the out-dated beta estimation techniques are employed when constructing their protfolios.

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