How well can Autoregressive Duration Models Capture the Price Durations Dynamics of Foreign Exchanges

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Abstract

Using a new omnibus density forecast evaluation procedure, we examine various commonly used Autoregressive Conditional Duration (ACD) models in capturing the price duration dynamics of Euro/Dollar and Yen/Dollar exchange rates. The ACD models under investigation include linear, logarithmic, Box-Cox, Exponential, Threshold, and Markov Switching ACD models with the exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. We find that none of the ACD models can adequately capture the full dynamics of foreign exchange rate price durations, either in sample or out of sample. However, some ACD models, particularly the Markov switching ACD model with Burr innovations, have not only the best in-sample fit, but also the best out-of-sample performance. We find that sophisticated nonlinear specifications for the conditional mean duration do not help much over linear ACD models in capturing the full dynamics of price durations, but the specification of the innovation distribution is important: the generalized Gamma or Burr distribution performs much better than the Weibull and exponential distributions. Moreover, it is important to relax the independence assumption for innovations and model higher order conditional moments of price durations.

Key Words: density forecast, full dynamics, foreign exchanges, out-of-sample, price duration, strong form ACD, weak form ACD

JEL Classification No: C4, C5, G0.

1 Introduction

The recent widespread availability of high-frequency financial transaction data has provided unprecedented opportunities to study various issues related to financial trading processes. A distinct feature of such data is that they arrive at irregular random time intervals. As emphasized in Goodhart and O'Hara (1997) and Madhaven (2000), the waiting time between intraday market events plays a key role for understanding the processing of private and public information in financial markets. In finance, the models of Diamond and Verrecchia (1987) and Easley and O'Hara (1992) provide theoretical justifications for developing time series models of inter-trade-arrival times. The autoregressive conditional duration (ACD) model, introduced by Engle and Russell (1998), is one of the most promising new econometric tools that focuses on the time intervals between the occurrences of trading events. It is tailor-made for the analysis of microstructure market issues and has been almost exclusively used to analyze high frequency financial data.

Following Engle and Russell (1998), a number of substantive extensions to the original linear ACD model have been made in the literature. Bauwens and Giot (2000) propose a logarithmic ACD model, which avoids the nonnegative constraints on model parameters implied by the linear ACD model. Dufour and Engle (2000) suggest Box-Cox and Exponential ACD models. Fernandes and Grammig (2003) develop a family of augmented ACD models, similar in spirit to the asymmetric GARCH models introduced by Hentschel (1995). Zhang, Russell and Tsay (2001) propose a threshold ACD model, which allows for different duration dynamics across different regimes. Other important ACD models include Markov chain regime switching ACD models (Hujer, Kokot and Vuletić 2003), smooth transition and time-varying ACD models (Meitz and Teräsvirta 2004), stochastic volatility duration models (Ghysels, Gourieroux and Jasiak 2004), and semiparametric ACD models (Hautsch 2002; Drost and Werker 2002).

In ACD modelling, besides the specification of the conditional mean duration, it is also important to specify the innovation distribution in order to capture the full dynamics of financial durations. Various innovation distributions have been used in the literature, including exponential and Weibull distributions, as used in Engle and Russell (1998), and Burr and generalized Gamma distributions, as used in Grammig and Maurer (2000) and Lunde (1999) respectively.

Given a wide variety of available ACD models, it is important to examine whether some ACD models perform better than others, and which type of model, if any, is particularly suited in capturing financial duration dynamics. The common practice to check adequacy of an ACD model has been using the Ljung-Box-Pierce type tests applied to estimated standardized or squared standardized durations of an ACD model. Unfortunately, the commonly used asymptotic chi-square distribution for this test is invalid, due to the complicated nontrivial impact of parameter estimation uncertainty (see Duchesne and Hong 2002 for more discussion). Several other specification tests for ACD models have also been proposed. They can be divided into three categories: (i) specification tests for the innovation distribution, (ii) specification tests for a conditional mean duration specification, and (iii) specification tests for the entire conditional probability density of an ACD model. Fernandes and Grammig (2000) suggest the tests of the first type. Assuming that the conditional mean duration model is correctly specified with i.i.d. innovations, they compare a nonparametric innovation density estimator with a model-implied parametric counterpart. Hautsch (2002) and Meitz and Teräsvirta (2004) advocate specification tests of the second type. Hautsch (2002) use conditional moment tests and integrated conditional moment tests, while Meitz and Teräsvirta (2004) propose Lagrange Multiplier (LM) type tests. Using Diebold, Gunther and Tay's (1998) density evaluation procedures, Dufour and Engle (2000) and Bauwens, Giot, Grammig and Veredas (2003) consider the tests of the third type. These procedures are easy to implement, and can provide hints to sources of model misspecification. However, they are informal and may not deliver a decisive conclusion about the relative performance of competing models. Moreover, the impact of parameter estimation uncertainty on the evaluation procedure is not considered. Dufour and Engle (2000) propose an alternative regression-based LM type test for density forecasts. However, the power of this test depends on the choice of instrumental variables which are somehow arbitrary.

In this paper, we use a recently developed omnibus test to examine whether commonly used ACD models can well forecast the probability density of foreign exchange price durations. The foreign exchange market is one of the most important financial markets in the world, with trading taking place 24 hours a day around the globe and trillions of dollars of different currencies transacted each day. Transactions in the foreign exchange market determine the rates at which currencies are exchanged, which in turn determine the costs of purchasing foreign goods and assets. Density forecasts are important in many applications. As argued by Diebold *et al.* (1998), Granger (1999) and Granger and Pesaran (2000), density forecasts are important for decision-making under uncertainty when forecasters' loss functions are asymmetric and the underlying process is non-Gaussian. In the present context, density forecasts for the price durations of exchange rates are particularly useful for many outstanding issues in international economics and finance. Price durations measure how long it takes for the price of an asset to move beyond a certain threshold. A trader might be interested in knowing this time interval as it could influence the speed with which he places an order. Price duration models are essentially a volatility model or more precisely the inverse of a volatility model (Engle and Russell 1997, 1998; Giot 2000) and thus play an important role in intra-day risk management. Density forecasts for price durations are crucial for the prediction of the price change intensity, which is important for valuing currency options and other currency derivatives. For example, Prigent, Renault and Scaillet (2001) offer an option pricing framework in incomplete markets by using a log ACD model to capture the full dynamics of a price duration process.

Hong and Li (2004) and Egorov, Hong and Li (2004) have recently developed nonparametric evaluation methods for a conditional density model. The test has an omnibus ability to detect a wide range of suboptimal density forecasts. Furthermore, it explicitly takes into account the impact of parameter estimation uncertainty on the evaluation procedure, an issue ignored in most existing evaluation procedures for density forecasts. We emphasize that the new test checks the entire conditional density of an ACD model rather than only the innovation distribution.

Applying the tests developed in Hong and Li (2004) and Egorov, Hong and Li (2004), we provide a comprehensive empirical analysis of both in-sample and out-of-sample performances of various commonly used ACD models for price durations of Euro/Dollar and Japanese Yen/Dollar exchange rates. The ACD models under examination include linear (LINACD) logarithmic (LOGACD), Box-Cox (BCACD), Exponential (EXPACD), Threshold (TACD), and Markov Switching (MSACD) models. For each model, we consider four commonly used innovation distributions—exponential, Weibull, generalized Gamma and Burr distributions. In contrast to earlier studies, we find that none of the ACD models can adequately capture the full dynamics of price durations of foreign exchanges, either in sample or out of sample, although the MSACD model with Burr innovations performs best. Sophisticated nonlinear specifications for the conditional mean duration (e.g., logarithmic, Box-Cox and Exponential forms) do not offer substantial improvement over linear ACD models in capturing the full dynamics of price durations of foreign exchanges. However, the specification of the innovation distribution is rather important: the exponential distribution always fits data poorly while the generalized Gamma distribution performs best (except that the Burr innovation performs best for the MSACD model). Moreover, to capture the full dynamics of price durations in foreign exchange markets, it is important to relax the i.i.d assumption for innovations and to model higher order conditional moments of price durations. Our results are similar for both Euro/Dollar and Yen/Dollar, and for both in-sample and out-of-sample.

In Section 2, we describe the evaluation procedures for the out-of-sample performance of a ACD model. A class of separate inference tests is also discussed, which can reveal useful information about where an ACD model is likely to be misspecified. In section 3, we review a variety of ACD models and discuss their relative merits. Section 4 describes the data and estimation results. Section 5 reports the in-sample and out-of-sample performances of the ACD models. Section 6 concludes.

2 Nonparametric Density Forecast Evaluation

Density forecasts have become a standard practice in many economic and financial applications. For example, modern risk control techniques often involve some form of density forecasts. In macroeconomics, monetary authorities in U.S. and U.K. (the Federal Reserve Bank of Philadelphia and the Bank of England) have been conducting quarterly surveys on density forecasts for inflation and output growth to help set their policy instruments (e.g., inflation target). There is also a growing literature on extracting density forecasts from options prices to obtain useful information on otherwise unobservable market expectations (e.g., Fackler and King 1990, Jackwerth and Rubinstein 1996, Soderlind and Svensson 1997, Ait-Sahalia and Lo 1998, Engle and Rosenberg 2002).

One of the most important issues in density forecasts is the evaluation of the quality of density forecasts (Diebold *et al.* 1998, Granger 1999). Suboptimal density forecasts for important macroeconomic variables, for example, may lead to suboptimal policy decisions (e.g., inappropriate level and timing in interest rate setting), which could have adverse consequence on resource allocations of an economy. In finance, suboptimal density forecasts may lead to misleading calculation of Value at Risk in risk management, and to large errors in derivatives pricing and hedging.

2.1 Nonparametric omnibus evaluation test

Evaluation of density forecasts is not trivial, since the probability distribution is not observable even *ex post*. Suppose $\{X_i, i = 1, 2, \cdots\}$ is a stationary time series with conditional density $p_0(x|I_{i-1})$, where $I_{i-1} = \{X_{i-1}, X_{i-2}, ...\}$ is the information available at time t_{i-1}. In our application, X_i will be the price duration of foreign exchange rates. For a given ACD model for X_i, there is a model-implied conditional density

$$\frac{\partial}{\partial x} P(X_i \leq x|I_{i-1}, \theta) \equiv p(x|I_{i-1}, \theta),$$

where $\theta \in \Theta$ is an unknown finite-dimensional parameter vector, and Θ is a parameter space. Suppose we have a random sample $\{X_i\}_{i=1}^T$ of size T, and we divide it into two subsets: an estimation sample $\{X_i\}_{i=1}^R$ of size R and a prediction sample $\{X_i\}_{i=R+1}^T$ of size $n \equiv T - R$. The former is used to estimate model parameter θ, and the latter is used to evaluate density forecasts. To evaluate density forecasts, we use the probability integral transform of X_i with respect to the model density, which is defined as follow:

$$Z_i(\theta) \equiv \int_{-\infty}^{X_i} p(x|I_{i-1}, \theta) \tag{1}$$

The transformed series $\{Z_i(\theta)\}$ can be called the "generalized residuals" of model $p(x|I_{i-1}, \theta)$. In an important work, Diebold *et al.* (1998) show that if model $p(x|I_{i-1}, \theta)$ is correctly specified in the sense that there exists some $\theta_0 \in \Theta$ such that $p(x|I_{i-1}, \theta_0)$ coincides with the true conditional density $p_0(x|I_{i-1})$, then the sequence $\{Z_i(\theta_0)\}$ should be i.i.d. $U[0,1]$. This characterization provides a convenient approach to evaluating $p(x|I_{i-1}, \theta)$. Intuitively, the $U[0,1]$ distribution indicates proper specification of the stationary distribution of X_i, and the i.i.d. property characterizes correct specification of its dynamics. If $\{Z_i(\theta)\}$ is not i.i.d. $U[0,1]$ for all $\theta \in \Theta$, then $p(x|I_{i-1}, \theta)$ is not optimal, and there exists room to improve $p(x|I_{i-1}, \theta)$. Thus, $p(x|I_{i-1}, \theta)$ can be evaluated by checking whether its generalized residuals is i.i.d. $U[0,1]$.

Most existing density forecast evaluation procedures examine the i.i.d. property and the uniformity property separately. While this is informative about possible sources of suboptimal density forecasts, it is preferable to use a single omnibus evaluation criteria that takes into account deviations from both i.i.d. and $U[0,1]$ jointly when comparing different models. Otherwise it may be difficult to decide which model is better in capturing the full dynamics of X_i if (e.g.) the generalized residuals of one model has less serial dependence but displays a more non-uniform distribution than the generalized residuals of another model.

Hong and Li (2004) developed a portmanteau in-sample evaluation procedure for a conditional density model, and Egorov, Hong and Li (2004) extend it to an out-of-sample context. They consider the impact of parameter estimation uncertainty and the choice of relative sample sizes between estimation and prediction samples on the evaluation procedure, two issues ignored by most existing evaluation procedures for density forecasts. They measure the distance between a density model and the true density by comparing a kernel estimator $\hat{g}_j(z_1, z_2)$ for the joint density of the pair of generalized residuals $\{Z_i(\theta), Z_{i-j}(\theta)\}$ with unity, the product of two $U[0,1]$ densities, where integer j is a lag order. The kernel estimator of the joint density of $\{Z_i(\theta), Z_{i-j}(\theta)\}$ is given by

$$\hat{g}_j(z_1, z_2) \equiv (n-j)^{-1} \sum_{t=R+j}^{T} \mathcal{K}_h(z_1, \hat{Z}_t)\mathcal{K}_h(z_2, \hat{Z}_{t-j}), \qquad j > 0, \tag{2}$$

where $\hat{Z}_i = Z_i(\hat{\theta}), \hat{\theta}$ is any $\sqrt{R}$-consistent estimator for θ_0, $\mathcal{K}_h(z_1, z_2)$ is a boundary-modified kernel function given by:

$$\mathcal{K}_h(x, y) \equiv \begin{cases} h^{-1}K(\frac{x-y}{h})/\int_{-(x/h)}^{1} K(u)du, & \text{if } \ x \in [0, h) \\ h^{-1}K(\frac{x-y}{h}), & \text{if } \ x \in [h, 1-h], \\ h^{-1}K(\frac{x-y}{h})/\int_{-1}^{(1-x)/h} K(u)du, & \text{if } \ x \in (1-h, 1], \end{cases} \tag{3}$$

and $K(\cdot)$ is a prespecified symmetric probability density, and $h \equiv h(n)$ is a bandwidth such that $h \to 0, nh \to \infty$ as $n \to \infty$. One example of $K(\cdot)$ is the quartic kernel $K(u) = 15/16(1 - u^2)^2\mathbf{1}(|u| \le 1)$, where $\mathbf{1}(\cdot)$ is the indicator function. We will use this kernel in our application. In practice, the choice of h is more important than the choice of $K(\cdot)$. Like Scott (1992), we choose

$h = \hat{S}_Z n^{-1/6}$, where $\hat{S}_Z$ is the sample standard deviation of $\{\hat{Z}_t\}_{t=R+1}^T$. This simple bandwidth rule attains the optimal rate for bivariate kernel density estimation.

The modified kernel in (2) automatically deals with the boundary bias problem associated with standard kernel estimation. The weighting functions in the denominators of the modified kernel $K_h(x, y)$ for x in the boundary regions $[0, h) \cup (1-h, 1]$ account for the asymmetric coverage and ensure that the kernel density estimator (3) is asymptotically unbiased uniformly over the support $[0, 1]$ of $Z_i(\theta)$.

Hong and Li (2004) propose an in-sample test based on a quadratic form between $\hat{g}_j(z_1, z_2)$ and 1, the product of two $U[0, 1]$ densities. This test has been extended to the out-of-sample context in Egorov, Hong and Li (2004):

$$\hat{Q}(j) \equiv \left[(n-j)h \int_0^1 \int_0^1 \left[\hat{g}_j(z_1, z_2) - 1\right]^2 dz_1 dz_2 - A_h^0 \right] / V_0^{1/2}, \qquad j = 1, 2, \cdots, \tag{4}$$

where the nonstochastic centering and scaling factors

$$\begin{aligned} A_h^0 &\equiv \left[(h^{-1} - 2) \int_{-1}^1 K^2(u) du + 2 \int_0^1 \int_{-1}^b K_b^2(u) du db \right]^2 - 1, \\ V_0 &\equiv 2 \left[\int_{-1}^1 \left[\int_{-1}^1 K(u+v) K(v) dv \right]^2 du \right]^2, \end{aligned}$$

and $K_b(\cdot) \equiv K(\cdot) / \int_{-1}^b K(v) dv$. Under suitable regularity conditions on the data generating process $\{X_i\}$, the model $p(x|I_{i-1}, t, \theta)$, the estimator $\hat{\theta}_R$, the kernel $K(\cdot)$, the bandwidth h, and the relative sizes n, R between estimation and prediction samples, $\hat{Q}(j) \to N(0, 1)$ in distribution when $p(x|I_{i-1}, \theta)$ is optimal.

The use of the $\hat{Q}(j)$ statistics with various lag orders can reveal the lag orders at which we have significant departures from i.i.d. $U[0, 1]$. To avoid the difficulty when one model has a smaller $\hat{Q}(j)$ at lag j_1 but another model has a smaller $\hat{Q}(j)$ at lag $j_2 \neq j_1$, Egorov, Hong and Li (2004) propose a portmanteau evaluation statistic

$$W(p) = \frac{1}{\sqrt{p}} \sum_{j=1}^p \hat{Q}(j). \tag{5}$$

For any given lag truncation order $p, W(p) \to N(0, 1)$ in distribution when $p(x|I_{i-1})$ is optimal. This may be viewed as a generalization of the popular Box-Pierce-Ljung autocorrelation-based portmanteau test from a linear time series context to a nonlinear time series context. It can check misspecification in a conditional density model of X_i, rather than only misspecification in a conditional mean model of X_i. As long as model misspecification occurs such that $\hat{Q}(j) \to \infty$ at some lag $j > 0$, we will have $W(p) \to \infty$ in probability. Therefore, $W(p)$ can be used as an omnibus procedure to evaluate density forecasts. Another important feature of $W(p)$ is that any

$\sqrt{R}$-consistent parameter estimator $\hat{\theta}_R$ suffices. It is not necessary to use an asymptotically most efficient estimator for θ. This is convenient, because asymptotically most efficient estimators such as maximum likelihood estimation (MLE) or approximated MLE may be difficult to obtain. One could choose a suboptimal but convenient estimator in implementing the test.

2.2 Diagnostic Tools

2.2.1 Diagnostics based on Generalized Model Residuals

When a density model is rejected by using $\hat{Q}(j)$ or $W(p)$, it would be interesting to explore possible reasons of the rejection. Diebold *et al.* (1998) illustrate how to use the histogram of $\{\hat{Z}_i\}$ and autocorrelogram in the powers of $\{\hat{Z}_i\}$ to reveal sources of model misspecification. Although informative, these graphical methods ignore the impact of parameter estimation uncertainty in $\hat{\theta}_R$ on the asymptotic distribution of evaluation statistics, which generally exists even when $R, n \to \infty$. Hong and Li (2004) provide a class of rigorous separate inference procedures that explicitly address the impact of parameter estimation uncertainty. This class of test statistics is defined as follows:

$$M_z(m,l) = \left[\sum_{j=1}^{n-1} k^2(j/p)(n-j)\hat{\rho}_{ml}^2(j) - \sum_{j=1}^{n-1} k^2(j/p)\right] \Big/ \left[2\sum_{j=1}^{n-2} k^4(j/p)\right]^{1/2}, \qquad m,l > 0, \tag{6}$$

where $\hat{\rho}_{ml}(j)$ is the sample cross-correlation between Z_t^m and Z_{t-j}^l. It is asymptotically $N(0,1)$ under correct model specification. Different choices of orders (m,l) examine various dynamic aspects of the underlying process $\{Z_t\}$. For example, the choice of $(m,l) = (1,1), (2,2), (3,3), (4,4)$ is sensitive to autocorrelations in level, volatility, skewness, and kurtosis of $\{Z_t\}$ respectively (see Diebold *et al.* 1998 for related discussion). Moreover, $M_z(1,2)$ and $M_z(2,1)$ can check ARCH-in-Mean and leverage effects respectively, which were not previously investigated in the density forecast evaluation literature.

2.2.2 Diagnostics based on Standardized Residuals

In ACD modeling, we have $X_i = \psi_i(\theta)\varepsilon_i$, where X_i is a duration process, $\psi_i(\theta)$ is a model for the conditional expected duration $E(X_i|I_{i-1})$, and ε_i is the innovation. Different insight can be obtained by examining the standardized residual $\hat{\varepsilon}_i = X_i/\psi_i(\hat{\theta}_R)$. For example, to see whether it is necessary to model higher order conditional moments of X_i, one can check whether $\{\varepsilon_i\}$ is i.i.d. To this end, we follow Hong and Lee (2003) and consider $\sigma_j(u,v) \equiv \mathrm{cov}(e^{\mathbf{i}u\varepsilon_i}, e^{\mathbf{i}v\varepsilon_{i-|j|}})$, the covariance function between $e^{\mathbf{i}u\varepsilon_i}$ and $e^{\mathbf{i}v\varepsilon_{i-|j|}}$. Straightforward algebra yields $\sigma_j(u,v) = \varphi_j(u,v) - \varphi(u)\varphi(v)$, where $\varphi_j(u,v) = E(e^{\mathbf{i}u\varepsilon_i + \mathbf{i}v\varepsilon_{i-|j|}})$ and $\varphi(u) = E(e^{\mathbf{i}u\varepsilon_i})$ are the joint and marginal characteristic functions of $(\varepsilon_i, \varepsilon_{i-|j|})$ respectively. Define the empirical measure

$$\hat{\sigma}_j(u,v) = \hat{\varphi}_j(u,v) - \hat{\varphi}_j(u,0)\hat{\varphi}_j(0,v), \quad j = 0, \pm 1, \pm 2, \cdots$$

where $\hat{\varphi}_j(u,v) = (n-|j|)^{-1} \sum_{i=|j|+1}^{n} e^{\mathbf{i}(u\hat{\varepsilon}_i + v\hat{\varepsilon}_{i-|j|})}$. We consider the following class of test statistics

$$\begin{aligned} M_\varepsilon(m,l) &= \left\{ \int \left[\sum_{j=1}^{n-1} k^2(j/p)(n-j)|\hat{\sigma}_j^{(m,l)}(u,v)|^2 \right] dW_1(u)dW_2(v) - \hat{C}_0^{(m,l)} \sum_{j=1}^{n-1} k^2(j/p) \right\} \\ &\quad \div \left[\hat{D}_0^{(m,l)}(1,l) \sum_{j=1}^{n-2} k^4(j/p) \right]^{1/2}, \end{aligned} \tag{7}$$

where the integer $m, l \geq 0, \hat{\sigma}_j^{(m,l)}(u,v) = \partial^{m+l}\hat{\sigma}_j(u,v)/\partial^m \partial^l, W_1(\cdot)$ and $W_2(\cdot)$ are positive and nondecreasing weighting functions, $k(\cdot)$ is a kernel function,[1] and the centering and scaling factors

$$\begin{aligned} \hat{C}_0^{(m,l)} &= \int \hat{\sigma}_0^{(m,m)}(u,-u)dW_1(u) \int \hat{\sigma}_0^{(l,l)}(v,-v)dW_2(v), \\ \hat{D}_0^{(m,l)} &= 2\int |\hat{\sigma}_0^{(m,m)}(u,u')|^2 dW_1(u)dW_1(u') \int |\hat{\sigma}_0^{(l,l)}(v,v')|^2 dW_2(v)dW_2(v'). \end{aligned}$$

Most commonly used kernels weigh down higher order lags. This is expected to enhance the power of the tests in practice, because financial markets are more influenced by recent events than by remote past events. We will use the Bartlett kernel $k(u) = (1-|z|)\mathbf{1}(|z| \leq 1)$ in our application. The lag order p can be chosen via suitable data-driven methods. Following Hong and Lee's (2003) proof, we can show that for each given pair of nonnegative integers (m,l),

$$M_\varepsilon(m,l) \longrightarrow N(0,1) \quad \text{in distribution}$$

under correct CAD model specification, provided lag order $p \equiv p(n) \to \infty, p/n \to 0$. We note that parameter estimation uncertainty in $\hat{\theta}_R$ has no impact on the asymptotic distribution of $M_\varepsilon(m,l)$.

The choice of (m,l) and $\{W_1(\cdot), W_2(\cdot)\}$ provide much flexibility in capturing various serial dependence of $\{\varepsilon_i\}$. For example, put $(m,l) = (0,0), W_1(\cdot) = W_2(\cdot) = W_0(\cdot)$, where $W_0 : \mathbb{R} \to \mathbb{R}^+$ is nondecreasing and weighs sets symmetric about 0 equally.[2] Then we obtain

$$\begin{aligned} M_\varepsilon(0,0) &= \left\{ \int \left[\sum_{j=1}^{n-1} k^2(j/p)(n-j)|\hat{\sigma}_j(u,v)|^2 \right] dW_0(u)dW_0(v) - \hat{C}_0^{(0,0)} \sum_{j=1}^{n-1} k^2(j/p) \right\} \\ &\quad \div \left[\hat{D}_0^{(0,0)}(1,l) \sum_{j=1}^{n-2} k^4(j/p) \right]^{1/2}, \end{aligned} \tag{8}$$

[1]The kernel $k(\cdot)$ here differs from the kernel $K(\cdot)$ used for probability density estimation.

[2]A commonly used example is $W_0(\cdot) = \Phi(\cdot)$, the N(0,1) cumulative distribution function (cdf).

where $\hat{C}_0^{(0,0)} = [\int \hat{\sigma}_0(u,-u)dW_0(u)]^2$ and $\hat{D}_0^{(0,0)} = 2[\int |\hat{\sigma}_0^{(0,0)}(u,u')|^2 dW_0(u)dW_0(u')]^2$. This checks generic serial dependence, which is useful in judging whether it is necessary to model higher order conditional moments of duration.

Suppose $\{\varepsilon_i\}$ is found to be serially dependent. It would be then interesting to examine the pattern of serial dependence in $\{\varepsilon_i\}$. To end this, we put $l = 0$, and use a Dirac $\delta(\cdot)$ function for $W_1(\cdot)$, with $W_2(\cdot) = W_0(\cdot)$.[3] Then

$$\begin{aligned} M_\varepsilon(m,0) &= \left\{ \int \left[\sum_{j=1}^{n-1} k^2(j/p)(n-j)|\hat{\sigma}_j^{(m,0)}(0,v)|^2 \right] dW_0(v) - \hat{C}_0^{(m,0)} \sum_{j=1}^{n-1} k^2(j/p) \right\} \\ &\quad \div \left[\hat{D}_0^{(m,0)} \sum_{j=1}^{n-2} k^4(j/p) \right]^{1/2}, \end{aligned} \tag{9}$$

where $\hat{C}_0^{(m,0)} = \hat{R}_m(0) \int \hat{\sigma}_0(v,-v)dW_0(v)$, $\hat{D}_0^{(m,0)} = 2\hat{R}_m^2(0) \int |\hat{\sigma}_0(v,v')|^2 dW_0(v)dW_0(v')$, and $\hat{R}_m(0)$ is the sample variance of $\hat{\varepsilon}_i^m$. Note that $\hat{\sigma}_j^{(m,0)}(0,v)$ is consistent for $\sigma_j^{(m,0)}(0,v) = \mathrm{cov}[(\mathbf{i}\varepsilon_i)^m, e^{\mathbf{i}v\varepsilon_{i-j}}]$. The choice of $m = 1$ checks the martingale hypothesis for $\{\varepsilon_i\}$. In particular, it has power against alternatives that have zero autocorrelation but a nonzero mean conditional on the ε_{i-j}, such as some bilinear and nonlinear moving average processes. This can reveal useful information whether a conditional mean duration model is adequate. Similarly, the choice of $m = 2, 3, 4$ can be used to test whether the conditional variance, skewness and kurtosis of ε_i are time-varying.

3 ACD Models

ACD models, first introduced by Engle and Russell (1998), are used to study the dynamics of arrival times between successive occurrences of trading events.

Let $X_i = t_i - t_{i-1}$ be the time intervals between two market events. Examples include the time between successive transactions, the time until a price change occurs or until a prespecified number of shares or level of turnover has been traded.[4] All ACD models can be embedded in the following framework:

$$\begin{cases} X_i = \psi_i \varepsilon_i, \\ \psi_i = E(X_i|I_{i-1}), \\ \varepsilon_i - 1 \sim \text{MDS with conditional pdf } f(\cdot|I_{i-1}), \end{cases} \tag{10}$$

By construction, the innovation ε_i is nonnegative, with $E(\varepsilon_i|I_{i-1}) = 1$ and conditional pdf $f(\cdot|I_{i-1})$. Because $\varepsilon_i = X_i/\psi_i$, ε_i is also called as a standardized duration. The specification of

[3] The Dirac delta function is defined as follows: $\delta(u) = 0$ if and only if $u \neq 0$ and $\int \delta(u)du = 1$.

[4] The price, volume and turnover duration processes can naturally be obtained from the trade duration series by dropping intervening observations from the sample, thus yielding a "thinned" or "weighted" duration process.

an ACD model includes: (i) ψ_i, the conditional mean duration, and (ii) $f(\cdot|I_{i-1})$, the conditional distribution of ε_i.

For most commonly used ACD models in the literature, $\{\varepsilon_i\}$ is assumed to be i.i.d. with marginal pdf $f(\cdot)$. This may be called a strong form ACD model, following the analogy of the strong form volatility model termed by Drost and Nijman (1993). For this class of ACD models, all past information enters the current duration through the conditional mean duration ψ_i. The dynamics of X_i is completely captured by ψ_i. Often, the assumption of the i.i.d. innovation is too strong to capture financial duration dynamics (e.g., Drost and Werker 2002). Again, by analogy with Drost and Nijman (1993), the case in which the demeaned innovation $\varepsilon_i - 1$ is an martingale difference sequences but not i.i.d. may be called a weak form ACD model, which allows for higher order dependence in durations. The flexibility of an ACD model lies in the rich host of candidates for the conditional pdf of ε_i as well as the functional form of the conditional mean ψ_i.

For a strong form ACD model, the conditional pdf of ε_i coincides with the marginal pdf of ε_i. In this case, several innovation distributions have been used in practice: the standard exponential, Weibull, generalized Gamma and Burr distributions:

(a) $f(\varepsilon_i) = \exp(-\varepsilon_i)$,

(b) $f(\varepsilon_i) = \gamma \left[\Gamma\left(1+\frac{1}{\gamma}\right)\right]^{\gamma} \varepsilon_i^{\gamma-1} \exp\left\{-\left[\Gamma\left(1+\frac{1}{\gamma}\right)\varepsilon_i\right]^{\gamma}\right\}, \quad \gamma > 0,$

(c) $f(\varepsilon_i) = \frac{\gamma\varepsilon_i^{\gamma\lambda-1}}{\Gamma(\lambda)} \left[\frac{\Gamma(\lambda+1/\gamma)}{\Gamma(\lambda)}\right]^{\gamma\lambda} \exp\left\{-\left[\frac{\varepsilon_i\Gamma(\lambda+1/\gamma)}{\Gamma(\lambda)}\right]^{\gamma}\right\}, \quad \lambda, \gamma > 0,$

(d) $f(\varepsilon_i) = \frac{\gamma}{c}\left(\frac{\varepsilon_i}{c}\right)^{\gamma-1}\left[1+\lambda\left(\frac{\varepsilon_i}{c}\right)^{\gamma}\right]^{-(1+\lambda^{-1})}, \qquad c = \frac{(\lambda)^{1+\gamma^{-1}}\Gamma(1+\lambda^{-1})}{\Gamma(1+\gamma^{-1})\Gamma(\lambda^{-1}-\gamma^{-1})}, \quad \gamma > \lambda > 0$

where $\Gamma(\cdot)$ is the Gamma function. Note that the generalized Gamma distribution reduces to the Weibull when $\lambda = 1$, and to the standard exponential when $\lambda = \gamma = 1$. When $\gamma < 1$, the Weibull distribution assigns a higher probability than the exponential distribution to extreme observations (very short and long durations). It also allows a non-flat hazard function, which is constant for the exponential distribution.[5] However, the Weibull hazard function is monotone: increasing if $\gamma < 1$, and decreasing if $\gamma > 1$. A more flexible hazard function can be obtained with the generalized Gamma distribution (e.g., Lunde 2000). It exhibits a nonmonotonic hazard function in certain regions of the parameter space: a $\cap$-shaped hazard when $\lambda\gamma > 1$ and $\gamma < 1$, and a U-shaped hazard when $\lambda\gamma < 1$ and $\gamma > 1$. Another distribution that has a hump-shaped hazard function and that nests the Weibull distribution is the Burr distribution. It is used in Grammig and Maurer (2000) to account for the stylized fact that the hazard function of some financial durations may be increasing for small durations and decreasing for long durations. We will consider these four innovation distributions to see which best describes the price duration of

[5]For a random variable X, its hazard function (or intensity function) is defined by $h(x) = f(x)/S(x)$, where $f(\cdot)$ and $S(\cdot)$ are the pdf and survival function of X, respectively. The survival function $S(x) \equiv P(X > x) = 1 - P(X \leq x),\ x > 0$.

foreign exchange rates.

Another key ingredient in an ACD model is the conditional mean duration $\psi(\cdot)$. We consider six most popular duration models: LINACD, LOGACD, BCACD, EXPACD, TACD and MSACD models. The first four belong to the class of strong form ACD models and the last two belong to the class of weak form ACD models. For a meaningful comparison of alternative ACD models and for simplicity, we follow Dufour and Engle (2000) and Bauwens *et al.* (2003) to limit the dynamic structure of the ACD models to the first lag order only.

3.1 Strong Form ACD models

3.1.1 Linear ACD models (LINACD)

Engle and Russell (1998) assume that ψ_i is a linear function of past durations and conditional durations, namely,

$$\psi_i = \omega + \alpha X_{i-1} + \beta\psi_{i-1}, \tag{11}$$

where $\omega > 0, \alpha \geq 0$ and $\beta \geq 0$, ensuring $\psi_i \geq 0$. This is analogous to a GARCH(1,1) model. It can account for duration clustering, a salient feature of financial high-frequency data. However, it has two main limitations, as pointed out by Engle and Dufour (2000). First, constraints on the parameters are needed to ensure that the linear model does not yield negative durations. When additional explanatory variables are added linearly to the model of (11), ψ_i may become negative, which is not admissible. Second, empirical evidence suggests (e.g., Engle and Russell 1998) that a nonlinear ψ_i may more accurately describe the dynamics of the conditional mean duration.

3.1.2 Logarithmic ACD models (LOGACD)

The limitations of LINACD have motivated Bauwens and Giot (2000) to introduce a LOGACD model:

$$\begin{aligned} \ln\psi_i &= \omega + \alpha\ln X_{i-1} + \beta\ln\psi_{i-1} \\ &= \omega + \alpha\ln\varepsilon_{i-1} + \beta'\ln\psi_{i-1} \end{aligned} \tag{12}$$

where $\beta' = \alpha + \beta$. While retaining the main characteristics of the LINACD model, this model is more flexible because no restrictions are required on the sign of its coefficients. Furthermore, for positive α, durations lower than the current conditional mean (so $\varepsilon_i = X_i/\psi_i < 1$) have a negative effect, while long durations ($\varepsilon_i > 1$) have a positive and marginally decreasing effect on the log of the expected duration. Thus the LOGACD model allows for nonlinear effects of short and long durations in the conditional mean, without requiring the estimation of additional parameters. However, it imposes a rigid adjustment process of the conditional mean to recent durations. For instance, because the logarithmic function asymptotically converges to minus infinity at zero, it is likely to have an overadjustment of the conditional mean after very short durations.

3.1.3 Box-Cox ACD models (BCACD)

To further improve upon the limitation of the LOGACD model, Dufour and Engle (2000) proposed a Box-Cox transformation ACD model

$$\ln \psi_i = \omega' + \alpha'(\varepsilon_{i-1}^{\delta} - 1)/\delta + \beta \ln \psi_{i-1} = \omega + \alpha \varepsilon_{i-1}^{\delta} + \beta \ln \psi_{i-1}. \tag{13}$$

This includes the LOGACD model as a special case (with $\delta \to 0$). The choice of an appropriate shock impact specification is data driven in this model (i.e., δ is estimated from data).

3.1.4 Exponential ACD models (EXPACD)

Dufour and Engle (2000) also introduce a class of EXPACD models similar in spirit to Nelson's (1991) EGARCH models. This allows for a piecewise linear news impact function kinked at the mean $E(\varepsilon_{i-1}) = 1$:

$$\ln \psi_i = \omega + \alpha \varepsilon_{i-1} + \delta \left| \varepsilon_{i-1} - 1 \right| + \beta \ln \psi_{i-1} \tag{14}$$

EXPACD models offer a captivating compromise between the need of greater flexibility and the burden of higher complexity. For standardized durations shorter than $E(\varepsilon_{i-1}) = 1$, it has a slope $\alpha - \delta$ and an intercept $\omega + \delta$; while for standardized durations longer than $E(\varepsilon_{i-1}) = 1$, the slope and intercept are $\alpha + \delta$ and $\omega - \delta$ respectively. It can capture asymmetric behaviors in price durations.

3.2 Weak form ACD models

3.2.1 Threshold ACD models (TACD)

Zhang, Russell and Tsay (2001) propose a TACD model that allows the conditional expected duration ψ_i to be nonlinear in past information variables. The TACD model is a simple but powerful generalization of the LINACD model, allowing different subregimes to have different conditional means and innovation distributions. Put $\mathbb{R}_j = [r_{j-1}, r_j), j = 1, \cdots, J$, for a positive integer J , where the γ_j, with $-\infty = r_0 < r_1 < \cdots < r_J = \infty$, are thresholds. The process $\{X_i\}$ follows a J-regime threshold ACD model if, when the threshold variable $Z_{i-d} \in \mathbb{R}_j$,

$$\begin{cases} \psi_i = \omega_j + \alpha_j X_{i-1} + \beta_j \psi_{i-1}, \\ \varepsilon_i \equiv X_i/\psi_i \sim f(\varepsilon_i; \theta_j), \end{cases} \tag{15}$$

where the delay parameter d is a positive integer. We allow parameter θ in the innovation distribution to be different across regimes. Exogenous variables (e.g., observable market characteristics, such as bid-ask spreads and volumes) as additional regressors and other functional forms of conditional mean (e.g. linear, logarithmic, Box-Cox and exponential forms) can also be used in (15). As documented in Zhang *et al.* (2001), there is a strong evidence that fast and slow transaction periods of NYSE stocks display different dynamics. We will examine whether this is also true of price durations of foreign exchanges. Here, we assume $Z_{i-d} = X_{i-1}$ and focus on a two-regime TACD model analogous to Zhang *et al.* (2003):

$$\begin{cases} \psi_i = \omega_j + \alpha_j X_{i-1} + \beta_j \psi_{i-1}, \\ \varepsilon_i^{(j)} \equiv X_i/\psi_i \sim f(\varepsilon_i; \theta_j), \end{cases} \quad \text{if } X_{i-1} \in \mathbb{R}_j, \ j = 1, 2, \tag{16}$$

where the parameters in both conditional expected duration ψ_i and innovation density $f(\cdot;\cdot)$ are allowed to vary across regimes. The innovation ε_i has a discrete mixture distribution. For a given regime $j, \{\varepsilon_i^{(j)}\}$ is i.i.d. Also, $\{\varepsilon_i^{(j)}\}$ and $\{\varepsilon_i^{(k)}\}$ are independent for $j \neq k$. However, the conditional distribution of ε_i given I_{i-1} is time-varying. In our empirical study, we will also employ a regime-adapted version of logarithmic forms for ψ_i.

3.2.2 Markov regime switching ACD models (MSACD)

The MSACD model, proposed by Hujer *et al.* (2002), allows the duration process X_i to depend on a latent state variable S_i that follows a Markov chain. This model nests many existing ACD models. It is closely related to Markov Switching autoregressive regression models popularized by Hamilton (1989) in econometrics. The introduction of the latent state variable S_i can be justified in the light of recent market microstructure theories. For instance, the unobservable regime can be associated with the presence (or absence) of private information about an asset's value that is initially available exclusively to a subset of informed traders and only eventually disseminates through the process of trading to the broader public of all market participants. Hujer *et al.* (2003) fit a MSACD model to the Boeing stock data on NYSE and show that it can capture several specific characteristics of intertrade durations while other ACD models fail. In our application to price durations of foreign exchanges, we assume that there are two regimes and the conditional mean duration ψ_i depends on the latent state variable S_i as follows:

$$\begin{cases} \ln \psi_i^{(S_i)} = \omega(S_i) + \alpha(S_i) \ln X_{i-1} + \beta(S_i) \ln \psi_{i-1}, \\ \varepsilon_i^{(S_i)} = X_i/\psi_i^{(S_i)} \sim f(\varepsilon_i; \theta(S_i)). \end{cases} \tag{17}$$

We refer to the regime in which $S_i = 1$ the first regime, and $S_i = 2$ the second regime. To avoid the computational intractability due to the dependence of the conditional mean ψ_i on the entire history of data, we follow Gray (1996) to average over all regime-specific conditional expectations according to

$$\psi_i = P(S_i = 1|I_{i-1})\psi_i^{(1)} + P(S_i = 2|I_{i-1})\psi_i^{(2)},$$

where $P(S_i = j|I_{i-1})$ is the probability that S_i is in state j given the filtration I_{i-1}. We assume a constant transition probability: $P(S_i = j|S_{i-1} = l) = p_{jl}$, where $j, l = 1, 2, .$ The associated conditional density for the price duration is given by

$$f(x|I_{i-1}) = P(S_i = 1|I_{i-1})f(x|S_i = 1, I_{i-1}) + P(S_i = 2|I_{i-1})f(x|S_i = 2, I_{i-1}),$$

where $P(S_i = j|I_{i-1})$, the *ex ante* probability that the data is generated from regime j at t_{i-1}, can be obtained by a recursive procedure described in Hamilton (1994).

By letting the parameters in the innovation distribution depend on the state variable S_i, the MSACD models allow for time-varying higher order conditional moments of X_i. TACD models are closely related to the MSACD models. Both of them belong to the class of discrete mixture models and allow the innovations to have time-varying conditional higher order moments. However, their mechanisms of regime determination are different: TACD models allow switches between different regimes to be driven by observable lagged dependent variables. It is interesting to examine the relative performance of these two classes of models in capturing the full dynamics of price durations of foreign exchanges.

4 Data and Estimation

We consider two intraday foreign exchange rates— Euro/Dollar and Yen/Dollar, from July 1, 2000 to June 30, 2001. Euro and Yen are two most important currencies in the world after the U.S. dollar. The launch of Euro has been probably the most important event in the history of the international monetary and financial system since the end of the Bretton Woods system in the early 1970s. It has created the world's second largest single currency area after U.S. In the foreign exchange market, Euro/Dollar is the busiest pair of currencies: it is estimated that 40 percent of the trading is between this pair, which is twice as large as the Dollar/DM pair had, and twice as large as the Yen/Dollar pair has. The Japanese economy was in a prolonged recession over the last decade, and as a result, the Yen/Dollar rate might have a different dynamics from the Euro/Dollar rate.

The data, obtained from Olsen & Associates, are indicative bid and ask quotes posted by banks. We choose the sample period between July 1, 2000 to June 30, 2001 to wait for the market to have stabilized after the introduction of Euro as a new currency in January 1, 1999 and to avoid the impact of "September 11" Incident. The foreign exchange market operates around the clock 7 days a week and the typical rate of quote arrivals differs dramatically on weekends and weekdays and between business hours in different countries and in different time zones (e.g, Goodhart and Figliuoli 1991, Bollerslev and Domowitz 1993, Engle and Russell 1997). To utilize days with a common typical pattern, only data on Wednesdays are used. This subsample consists of 52 days, and 1,264,553 observations on Euro/Dollar and 623,687 observations on Yen/Dollar.

Although we focus on price durations, it is useful to first examine quote arrival rates and quote durations. For our data, a typical weekday has almost 24,300 and 16,000 quote arrivals, and on average, a quote arrives every 3.5 s and 5.5 s for Euro/Dollar and Yen/Dollar respectively, where "s" denotes the unit of second. Figure 1 plots the histogram of raw durations up to 30 s, showing that two exchange rates share a similar pattern. The majority of quotes (about 86% for Euro/Dollar and 67% for Yen/Dollar) arrive within 5 s of the previous quote. Clearly, the Euro/Dollar market is more active than the Yen/Dollar market.

There is a strong seasonality in quote durations as a deterministic function of the time of the day. It is important to deseasonalize raw duration data. Following Engle and Russell (1997), we regress the duration on a pure time-of-day to obtain a consistent estimator of the typical duration shape due to the time-of-day effect. Dividing durations by their estimated typical shape thus gives "seasonally adjusted" durations. Figure 2 presents the predicted duration as a deterministic function of the time of day. This was obtained by regressing the observed duration on 96 time-of-day dummy variables, each per 15 minute time interval. Both Euro/Dollar and Yen/Dollar display the same pattern of seasonality as have been revealed in previous studies of quote frequency (Bollerslev and Domowitz 1993, Engle and Russell 1997). Trading activity picks up after midnight as the Asian Pacific markets such as Tokyo, Sydney, Singapore and Hong Kong open. The abrupt decline in arrivals between hours 3:00–4:00 GMT signals lunchtimes in these markets. We find most quote activity between 5:00 and 16:00 GMT in the afternoon Far Eastern trading session and during the overlap of the New York and European markets. During this period quotes arrive at a rate of about 1 trade every 2 s for Euro/Dollar and 4 s for Yen/Dollar on average. Activity declines after the New York market closes and before the Far Eastern markets open again.

Price durations are the time needed to witness a given cumulative change in the price. They are usually defined on the mid-point of the bid-ask quote process. Such definition is one of the favorite ways used in the literature to thin the quote point process (e.g., Bauwens and Giot 2000, 2003; Engle and Russell 1997, 1998). It has several advantages. First, the bid-ask bounce can be avoided. In a dealer's market, the bid-ask bounce can be annoying to work with, as it is a main feature of data but gives little information. Second, as the minimum amount of time for the price to increase or decrease by at least c, a predefined threshold, price durations are closely linked to the instantaneous volatility of the mid-quote price process. Therefore, we can investigate the determinants of price volatility by adding exogenous and lagged dependent variables to an ACD model of price durations. Thirdly, as documented in Engle and Russell (1997, 1998) for the foreign exchange market and the New York Stock Exchanges or in Biais, Hillion and Spatt (1995) for the Paris Bourse, the quotes process is often characterized by a short term transitory component that gives little information about the value of the asset; while significant movement of the mid-point often reflects some informative events.

Define the "midprice" at time t_i as $P_i = (bid_i + ask_i)/2$, where bid_i and ask_i are the current bid and ask prices associated with transaction time t_i. A threshold c characterizes price changes. If $c = 0$, we would count every single movement in the midpoint as a price change. To better capture movements in the price at which transactions occur, we will choose a positive value of c. Figure 3 displays the histogram of the spreads. Most spreads are 0.0005 for Euro/Dollar and 0.05 for Yen/Dollar respectively, which both account for more than 40% respectively. We thus set $c = 0.0005$ and 0.05 for Euro/Dollar and Yen/Dollar respectively. These choices of c yield a sample size of 20,584 for Euro/Dollar and 15,818 for Yen/Dollar. Table 1 gives some descriptive statistics. For Euro/Dollar, the minimum price duration is 1 s, the maximum is 12,666 s (about 3.5 hours), and the average is nearly 214 s. For Yen/Dollar, the minimum price duration is 1 s, the maximum is 12,697 s, and the average is nearly 276 s, with the last two larger than those of Euro/Dollar. Figure 4 presents the histogram for the price durations, whose patterns differ a bit from those of raw quote durations. Most common price durations are 1 s, accounting for 7.3% for Euro/Dollar and 3.6% for Yen/Dollar respectively.

Figure 5 presents the seasonalities for price durations, which are calculated in the same manner as for quote durations. The two kinds of seasonality show similar patterns. The price durations are on average about once every 100 s for Euro/Dollar and 160 s for Yen/Dollar between 6:00 and 15:30 GMT. When the European market closes, price changes occur much less frequently. They become as infrequent as roughly once every 1000s and 830s for Euro/Dollar and Yen/Dollar respectively around 22:30 GMT. However, Yen/Dollar price changes occur most frequently around 24:00 GMT when the Asian markets open again.

We adjust price durations by filtering out the seasonalities as follows:

$$X_i = Y_i / g(t_i),$$

where $\{Y_i\}$ is an original price duration, and $g(\cdot)$ is the seasonal effect on price durations. The mean of the deseasonalized series X_i is approximately unity. The standard deviations are 1.71 and 1.63 for the "seasonally adjusted" Euro/Dollar and Yen/Dollar price durations respectively, indicating overdispersion. The adjusted price duration process in Figure 6 looks stationary. Prices tend to experience periods of rapid and slow movement respectively, displaying strong price duration clustering. There are a few jumps or outliers as well. Figure 7 shows that adjusted price durations have a decreasing pdf. A striking feature of price durations is the presence of autocorrelation even after removing the time-of-day effects. Figure 8 reports the first 50 sample autocorrelations of two adjusted price durations. The slow decaying autocorrelations indicate persistent price duration clustering. Ljung-Box test statistics with 15 lags are 34,719 and 30,558 for Euro/Dollar and Yen/Dollar respectively, implying rather significant autocorrelation in price durations.

Following the usual practice in the literature, we focus on the adjusted series X_i rather on the original series Y_i. For out-of-sample evaluation, we divide the data into two equal halves. The first half (10,292 observations for Euro/Dollar and 7,909 observations for Yen/Dollar) is used for estimation and the second half is used for out-of-sample evaluation. For both Euro/Dollar and Yen/Dollar, durations in two subsamples appear to have similar characteristics. We consider six classes ACD models: LINACD, LOGACD, BCACD, EXPACD, TACD and MSACD, combined with four innovation distributions—the exponential, Weibull, generalized Gamma and Burr distributions.[6] This generates twenty four ACD models, all of which are estimated via MLE. The optimization algorithm is the well-known BHHH with STEPBT for step length calculation and is implemented via the constrained optimization code in GAUSS Window Version 5.0. For TACD models, the MLE estimation is performed by a grid search over threshold values r and by maximizing the likelihood function given r. Parameter estimates of various ACD models are reported in Table 2.

5 Empirical Evidence

We now use Hong and Li's (2004) test to evaluate various ACD models for the price durations of foreign exchanges. The performance of each model is measured by the $W(p)$ statistic, reported in Panels A and B of Table 3. For space, we only report $W(5), W(10)$ and $W(20)$.[7]

5.1 In-sample Performance

We first evaluate in-sample performance of ACD models. Based on parameter estimates in Table 2, we calculate in-sample generalized residuals $\{Z_i(\hat{\theta})\}_{t=1}^{R}$ in (1), where R is the size of in-sample observations. Although some models perform better than others, $W(p)$ in Panel A of Table 3 overwhelmingly rejects all ACD models at any conventional significance level. In other words, none of the ACD models adequately captures the full dynamics of price durations for Euro/Dollar and Yen/Dollar. Our results differ from Bauwens, Giot and Veredas (2003), who find that LOGACD models based on generalized Gamma or Burr innovations perform satisfactorily for some stock price durations.[8] Among all ACD models, the MSACD model with the Burr distribution performs best for both Euro/Dollar and Yen/Dollar, with $W(5)$ statistics around 30

[6]For TACD and MSACD, we assume that the innovations in a specific regime follows exponential, Weibull, generalized Gamma and Burr distributions respectively, with different parameters across different regimes. The marginal distribution of innovations is different from their conditional distribution in both TACD and MSACD models.

[7]The results of $W(p)$, for $p = 1, 2, \cdots, 30$, are available from the authors upon request.

[8]One possible reason is that the evaluation tools used are different. Another possibility is that the price durations of stocks have a different dynamics from the foreign exchange rates.

and 20 respectively. This is in line with the results of Hujer *et al.* (2003) that MSACD models have a better in-sample fit than other ACD models. With the same innovation distribution, LINACD, LOGACD, BCACD, EXPACD and even TACD models in some cases perform rather similarly, although the TACD models are more sophisticated. This implies that nonlinear ACD models for ψ_i do not always outperform the LINACD model. In other words, a linear model for ψ_i performs as well as commonly used nonlinear ACD models for ψ_i in capturing the full dynamics of price durations of foreign exchanges. Among four innovation distributions, the exponential distribution always fits poorly while the generalized Gamma distribution performs best (the Burr innovation performs best for the MSACD model). For the ACD models with the exponential innovations (except the MSACD model), $W(5)$ statistics are extremely large—over 3,000 for Euro/Dollar and over 1,000 for Yen/Dollar. $W(10)$ and $W(20)$ tell the same story as $W(5)$. The $W(p)$ statistics imply that sophisticated specifications of the conditional mean duration ψ_i do not help much in capturing the full dynamics of price durations. However, the specification of the innovation distribution is important: either the generalized Gamma or Burr distribution always performs better. Moreover, it is rather important to relax the i.i.d. assumption for the innovation and consider higher order conditional moments of X_i. We find that the relative rankings among all ACD models are generally similar for two foreign exchange rates. The $W(p)$ statistics for Yen/Dollar are only about half that of Euro/Dollar, indicating that these models can better describing the price duration dynamics of Yen/Dollar. This may be due to the fact that Euro/Dollar is more active than Yen/Dollar and its price durations display more time-varying clustering and dispersion. As a consequence, it is more difficult to capture the price duration dynamics for Euro/Dollar than for Yen/Dollar.

Below, we investigate possible sources for the failure of ACD models by separately examining the uniform distribution and the i.i.d. properties of the generalized residuals of each model. Figures 9-1 and 9-2 display the histogram of the generalized residuals in (1). Consistent with the $W(p)$ statistics, we find that the marginal density of the generalized residuals of MSACD models are much closer to the uniform distribution than other ACD models. In particular, the distribution of the generalized residuals of the RSACD model with the Burr innovations is the closest to the uniform distribution. Given the same innovation distribution, the generalized residuals of LINACD, LOGACD, BCACD, EXPACD and TACD models have similar marginal distributional shapes. The density estimates of the generalized residuals of all ACD models (except RSACD) with the exponential innovation always exhibits a U shape, with pronounced peaks at both ends, especially at the left end. This indicates that the exponential innovation distribution underpredicts the tails of price durations, particularly extremely short price durations. In contrast, the generalized residuals of the ACD models with Weibull, generalized Gamma and Burr distributions usually show a similar $\cap$-shape density, implying that these models turn to overpredict ultra-long and ultra-short price durations. Both Euro/Dollar and Yen/Dollar ACD models often tell the same story.

We now examine the performance of each model in capturing various specific dynamics of price durations. First, we check a pattern of serial dependence of the generalized residuals $\{Z_i(\hat{\theta}_R)\}$ of each ACD model for two exchange rates. Panels A and B of Table 4 report the diagnostic tests $M_z(m, l)$ for the in-sample generalized residuals. Almost all $M_z(m, l)$ statistics are rather significant at the 5% level and most of them are significant at the 1% level, indicating that there exists neglected dynamic structure in price durations for all ACD models. With the same innovation distribution, LOGACD, BCACD and EXPACD often exhibit similar performances and perform a little better than LINACD. Weak form ACD models perform better than strong form ACD models. Among four innovation distributions, the exponential distribution always delivers a larger $M_z(m, l)$, with $M_z(0, 0)$ around 100. This may be due to the fact that the exponential distribution cannot capture overdispersion well in price durations. The generalized Gamma and Burr distributions always perform better in capturing price duration dynamics. In particular, the generalized Gamma distribution performs best in most cases. The models— TACD models with Burr innovations, RSACD models with generalized Gamma innovations, and RSACD models with Burr innovations— have relatively small $M_z(m, l)$ statistics, with $M_z(0, 0)$ around 10. For Euro/Dollar, the TACD model with Burr innovations delivers the smallest $M_z(0, 0)$, which is 8.18; for Yen/Dollar, the RSACD model with the Burr innovations delivers the lowest $M_z(0, 0)$, which is 4.27. These results show that relaxing the i.i.d. assumption for the innovation helps a lot in capturing the price duration dynamics.

Next, we check the independence assumption for the innovations $\{\varepsilon_i\}$. Table 5 reports the $M_\varepsilon(m, l)$ statistics for estimated standardized model residuals $\{\hat{\varepsilon}_i = X_i/\hat{\psi}_i\}$. The $M_\varepsilon(0, 0)$ statistics in Panels A and B of Table 4 are all larger than 15.0, indicating strong serial dependence in $\{\hat{\varepsilon}_i\}$. This apparently contradicts the independence assumption for the innovation. The $M_\varepsilon(1, 0)$ and $M_\varepsilon(2, 0)$ statistics are significant at the 5% level for almost all ACD models, indicating that there exists neglected dynamic structure in both conditional mean and conditional dispersion of price durations. For Euro/Dollar, $M_\varepsilon(3, 0)$ and $M_\varepsilon(4, 0)$ test statistics are not significant (except very few cases) at the 5% level. For Yen/Dollar, while the $M_\varepsilon(3, 0)$ statistics are small than $M_\varepsilon(1, 0)$ and $M_\varepsilon(2, 0)$, they are significant at the 5% level in most cases. In contrast, $M_\varepsilon(4, 0)$ statistics are not significant at the 5% level. These results imply that the price duration may have strong serial dependence in its first two conditional moments (i.e., conditional mean duration and conditional dispersion of durations), but it may have rather weak or nonexistent higher order serial dependence.

Similar to Hujer *et al.* (2003), our analysis shows that the MSACD models, particularly the RSACD model with Burr innovations, have the best in-sample performance. Diagnostic analysis shows that RSACD models better capture the stationary density of the price duration. The weak form ACD models, such as TACD and RSACD, are better than strong form ACD models in capturing serial dependence of the generalized residuals. Moreover, both the portmanteau test $W(p)$ and various diagnostic tests $M_z(m,l)$ and $M_\varepsilon(m,l)$ suggest that the ACD models with generalized Gamma or Burr innovations can better capture the in-sample price duration dynamics of foreign exchanges.

5.2 Out-of-Sample Density Forecast Performance

Next, we study out-of-sample performance of ACD models. We are interested in checking whether the ACD models that have the best in-sample performance also have the best out-of-sample performance. Using parameters estimated in Table 2, we obtain out-of-sample generalized residuals $\{Z_t(\hat{\theta})\}_{t=R+1}^{T}$ for Euro/Dollar and Yen/Dollar respectively. Interestingly, the out-of-sample performances of ACD models are similar to their in-sample ranking. The out-of-sample $W(p)$ statistics in Panel B of Table 3 are significant at the 1% level, indicating that none of the ACD models can adequately forecast the full dynamics of price durations. Again, the RSACD model with Burr innovations, which has the best in-sample fit, also has the best out-of-sample forecast, with $W(5)$ equal to 29.5 and 17.0 for Euro/Dollar and Yen/Dollar respectively. With the same innovation distribution, LINACD, LOGACD, BCACD, EXPACD and TACD models perform rather similarly. This indicates that sophisticated nonlinear modeling for the conditional mean ψ_i helps little in improving out-of-sample density forecasts of price durations. However, the innovation distribution specification is important: the exponential distribution often has the worst performance, with overwhelming large $W(p)$ statistics. The Weibull distribution helps a lot in improving density forecasts: the $W(p)$ statistics decrease from over 1000 for the exponential distribution to well below 500, given the same conditional mean specification ψ_i. Generalized Gamma and Burr innovations often have the best performances. This is similar to the results of Dufour and Engle (2000) that the choice of the innovation distribution becomes critical when forecasting the trade duration density. We also find that it is important to relax the i.i.d. assumption for innovations. Taking into account higher order conditional dependence via regime shifts and threshold principles helps a lot in forecasting the full dynamics of price durations.

To gauge possible sources of model misspecification, we next separately examine the uniform distribution and i.i.d. properties of out-of-sample generalized residuals. Figures 10-1 and 10-2 display histograms of out-of-sample generalized residuals of Euro/Dollar and Yen/Dollar. Consistent with the $W(p)$ statistics, we find that the marginal densities of out-of-sample generalized residuals of RSACD models are much closer to the uniform distribution than those of other models. In particular, the density of the out-of-sample generalized residuals of the RSACD model with Burr innovations is the closet to the uniform distribution. Given the same innovation distribution, the density of the out-of-sample generalized residuals of LINACD, LOGACD, BCACD, EXPACD and TACD models perform rather similarly. Figure 10 shows that the density estimates of the out-of-sample generalized residuals of all ACD models (except RSACD) with exponential innovations exhibit a U shape, with pronounced peaks at two ends, especially at the left end. This pattern is similar to the in-sample pattern. However, this situation has been greatly improved in the RSACD model with exponential innovations. The ACD models with Weibull, generalized Gamma and Burr innovations all have an ∩-shape density for their out-of-sample generalized residuals: more realizations than predicted fall into the left and right ends. In most cases, the generalized Gamma distribution slightly outperforms the Burr distribution, which in turn outperforms the Weibull distribution. Although the RSACD model with Burr innovations is the closet to the uniform distribution, it still slightly underpredicts the very short durations and overpredicts the short durations. This implies that all ACD models cannot fully account for the tail of price durations. However, the RSACD models, which allow for different regimes and higher order serial dependence, can better capture the fat tail of price durations.

While the RSACD model with Burr innovations characterizes the marginal density of the out-of-sample generalized residuals well, it still has difficulty in capturing various aspects of price duration dynamics, as can be seen from the $M_z(m,l)$ and $M_\varepsilon(m,l)$ statistics in Tables 4 and 5. Panels C and D of Table 4 reports $M_z(m,l)$ statistics for out-of-sample generalized residuals. All ACD models fail to satisfactorily capture serial dependence in the conditional mean, variance, skewness and kurtosis of their out-of-sample generalized residuals, with $M_z(m,l)$ statistics significant at any conventional significance level. In general, the weak form ACD models (TACD and RSACD) with generalized Gamma and Burr innovations perform better, giving smaller $M_z(0,0)$ statistics. There is a little difference from the in-sample case: the TACD (rather than the MSACD) model with Burr innovations has the smallest $M_z(0,0)$ statistic for both Euro/Dollar and Yen/Dollar. For Euro/Dollar, the RSACD model with Burr innovations has a larger $M_z(0,0)$ than many other models (e.g., RSACD and BCACD models with generalized Gamma innovations). This implies that relaxing the i.i.d. assumption for the innovation, allowing regime shifts, and using generalized Gamma or Burr innovations can significantly improve forecasting the full dynamics of price durations of foreign exchanges. The $M_\varepsilon(m,l)$ tests for the standardized residuals are reported in Panels C and D of Table 5. The empirical results are similar to that of the in-sample case: for both Euro/Dollar and Yen/Dollar, $M_\varepsilon(0,0), M_\varepsilon(1,0)$ and $M_\varepsilon(2,0)$ is rather large, while $M_\varepsilon(3,0)$ and $M_\varepsilon(4,0)$ are relatively small.

In summary, our analysis shows that none of commonly used ACD models can adequately capture the full dynamics of price durations of foreign exchanges, either in-sample or out-of-sample. This differs from Bauwens *et al.* (2003), who use different evaluation methods and find that LINACD and LOGACD models with generalized Gamma and Burr innovations perform satisfactorily for stock price durations. However, some ACD models outperform others. The RSACD models with Burr innovations have not only the best in-sample fit but also the best out-of-sample performance, which is consistent with the empirical results of Hujer *et al.* (2003) for stock transaction durations. Generally speaking, each ACD model has similar in-sample and out-of-sample performances. In particular, the ACD models that have best in-sample fit usually have best out-of-sample density forecasts. It seems that sophisticated nonlinear specifications for conditional expected durations do not help much in capturing the full dynamics of price durations. However, the specification of the innovation distribution is important: the exponential distribution always fits poorly while generalized Gamma and Burr distributions perform much better. Moreover, relaxing the i.i.d. assumption for the innovation, allowing higher order dependence in price durations, and taking into account possible regime shifts can help a lot in improving the performance of ACD models. Nevertheless, there seems to be a long way to find an adequate ACD model for the full dynamics of price durations of foreign exchanges, which remains to be an important research topic in the literature.

6 Conclusion

In high-frequency financial econometrics, price duration dynamics is important due to its close links to market microstructure theory, options pricing, and risk management. Applying Hong and Li's (2004) nonparametric portmanteau test for time series conditional distributional models, we provide a relatively comprehensive empirical study on in-sample and out-of-sample performances of a wide variety of ACD models in capturing the full dynamics of price durations of two exchange rates—Euro/Dollar and Yen/Dollar.

We find that none of the ACD models can adequately capture the price duration dynamics of Euro/Dollar and Yen/Dollar, either in-sample or out-sample. However, some ACD models, particularly the Markov switching ACD model with Burr innovations, have not only the best in-sample fit, but also the best out-of-sample performance. We find that sophisticated models for the conditional mean duration do not help much in capturing the full dynamics of price durations of foreign exchanges, but the specification of the innovation distribution is important: generalized Gamma or Burr distribution performs much better than Weibull and exponential distributions. The latter often performs poorest. Moreover, the conditional mean duration alone cannot fully capture the dynamics of price durations of foreign exchanges. It is important to relax the i.i.d assumption for the innovation, to model higher order conditional moments, and to allow possible regime shifts in price durations. Our findings are similar for both Euro/Dollar and Yen/Dollar and for both in-sample and out-of-sample.

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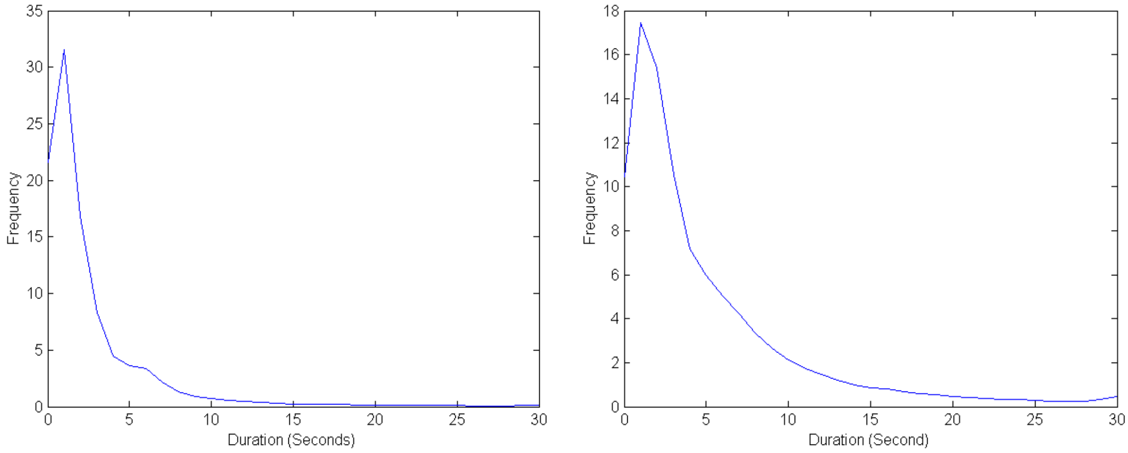
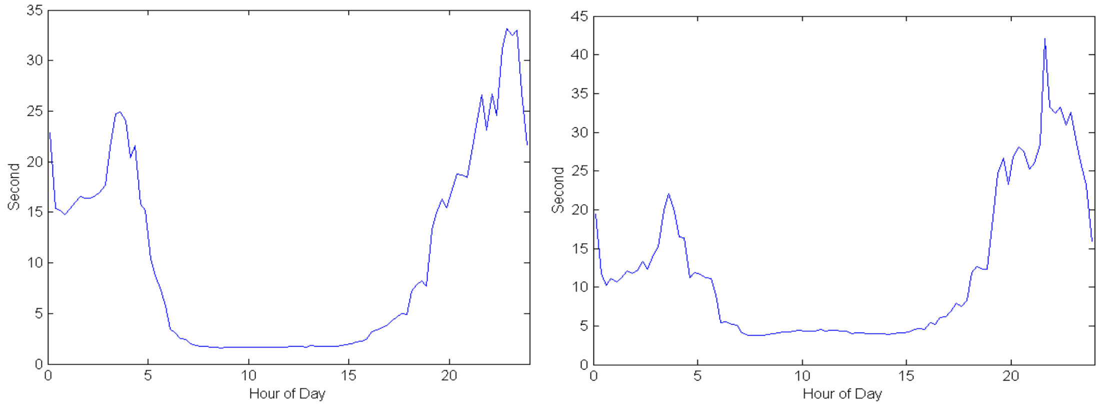
Figure 1. Histograms of raw quote durations



Euro/Dollar exchange rate

Yen/Dollar exchange rate

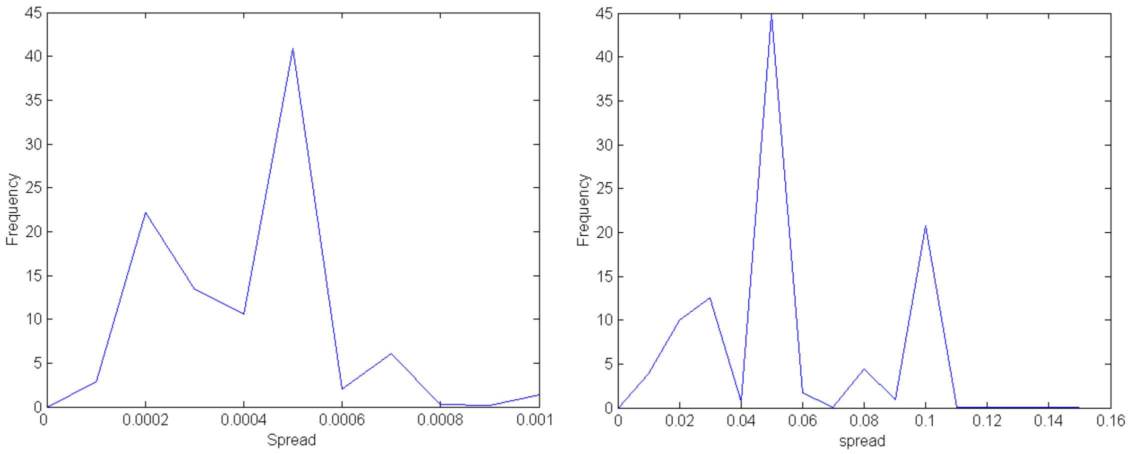
Figure 2. Expected quote durations conditional on time of day



Euro/Dollar exchange rate

Yen/Dollar exchange rate

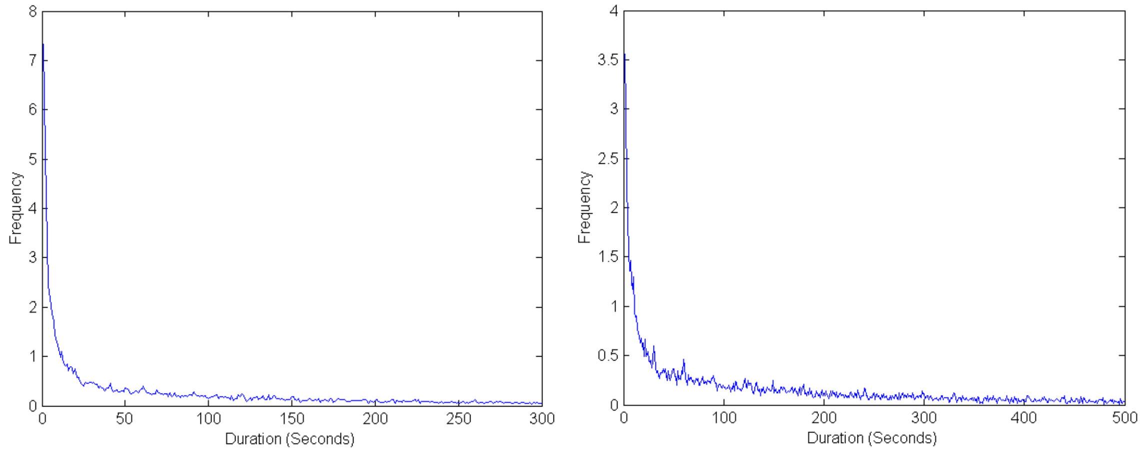
Figure 3. Histograms of observed spreads



Euro/Dollar exchange rate

Yen/Dollar exchange rate

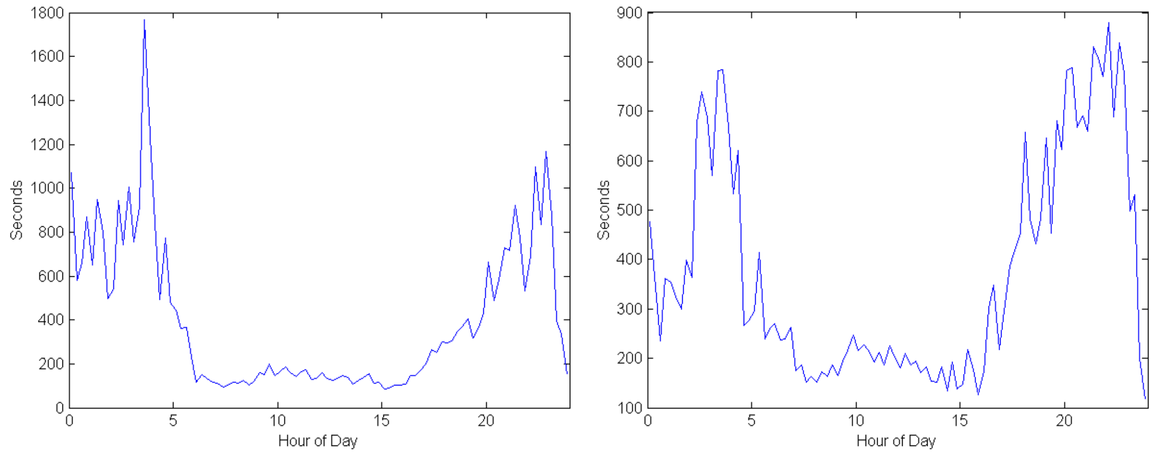
Figure 4. Histograms of raw price durations



Euro/Dollar exchange rate

Yen/Dollar exchange rate

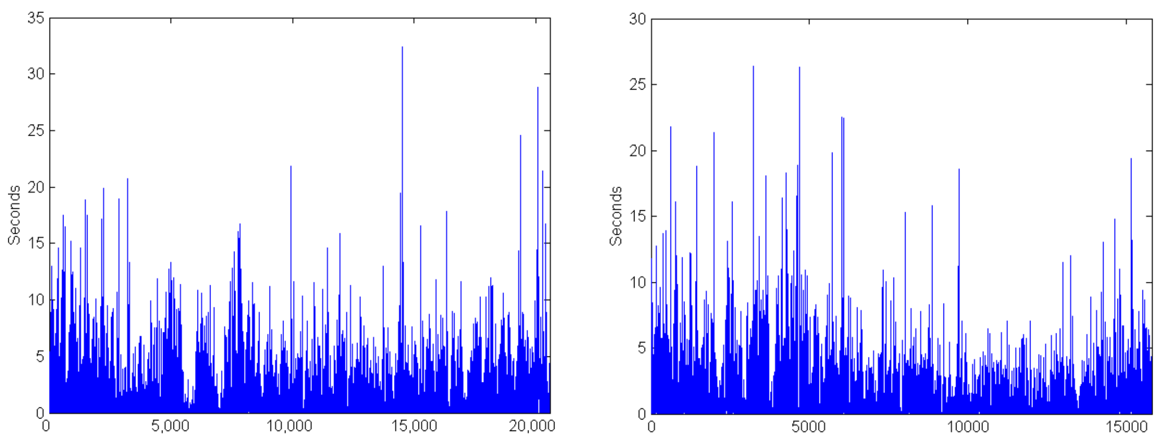
Figure 5. Expected price durations conditional on time of day



Euro/Dollar exchange rate

Yen/Dollar exchange rate

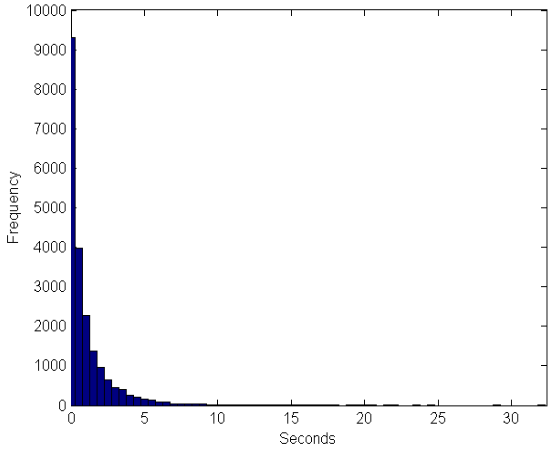
Figure 6. Seasonally adjusted price duration



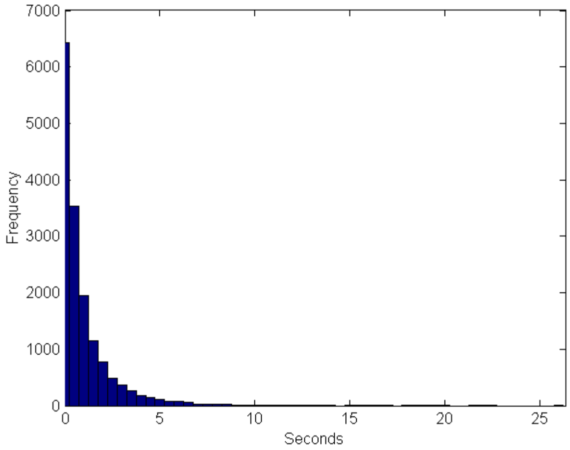
Euro/Dollar exchange rate

Yen/Dollar exchange rate

Figure 7.   Histogram of seasonally adjusted price duration

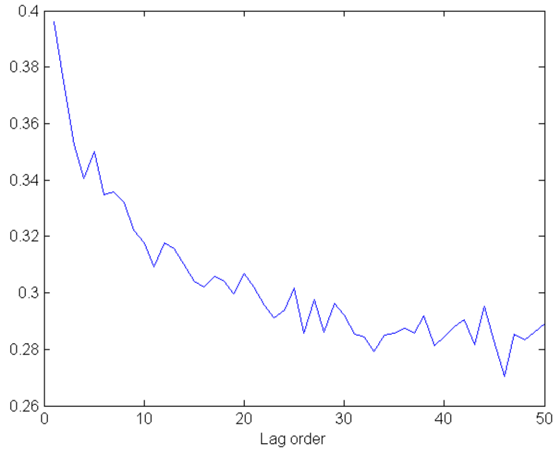


Euro/Dollar exchange rate

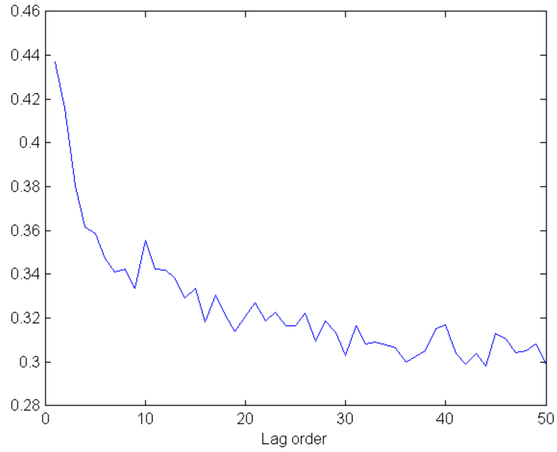


Yen/Dollar exchange rate

Figure 8.   Sample autocorrelations of seasonally adjusted price duration

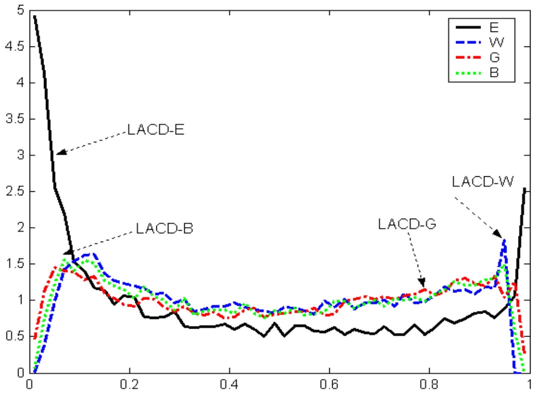


Euro/Dollar exchange rate

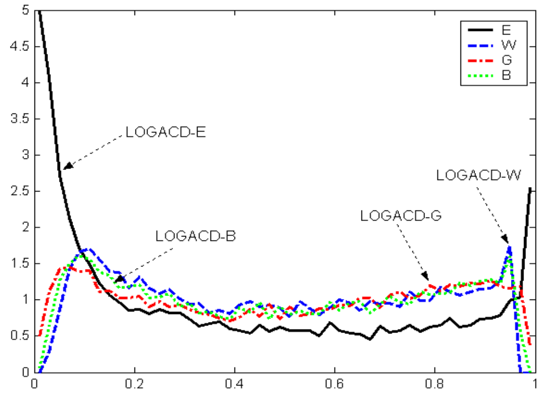


Yen/Dollar exchange rate

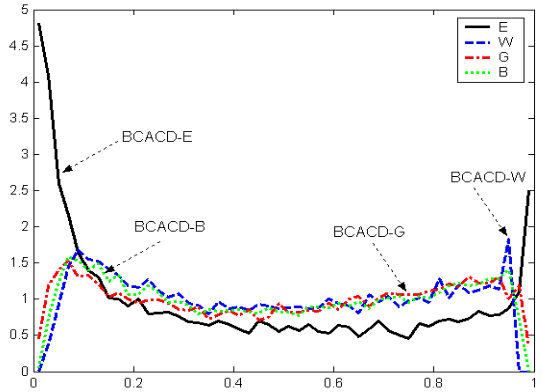
Figure 9-1. Histogram of in-sample generalized residuals of Euro/Dollar



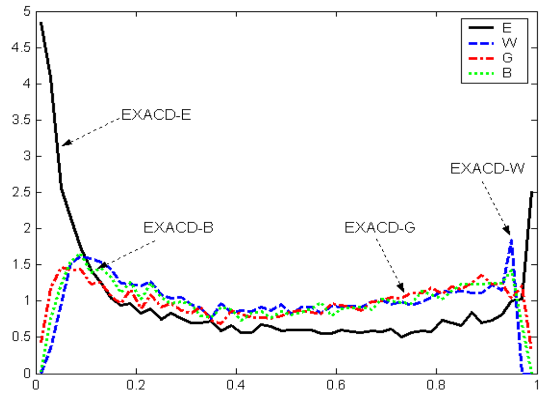
Linear ACD



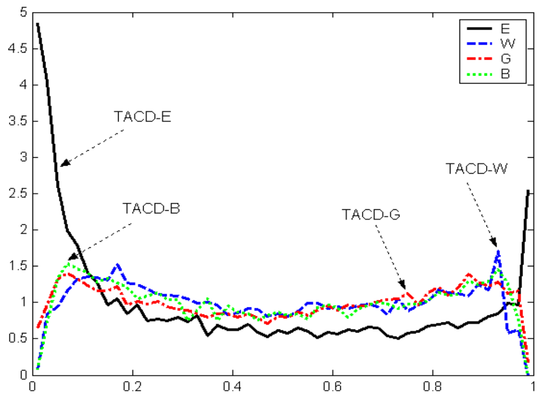
Log ACD



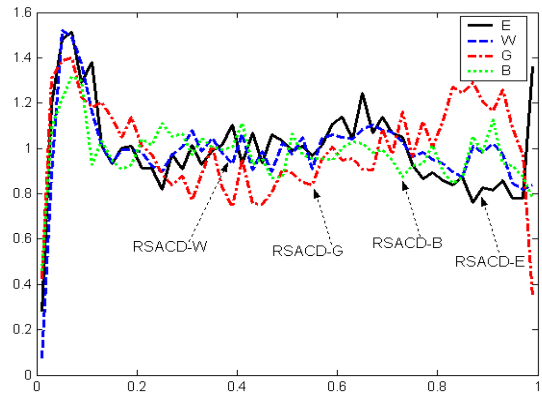
Box-Cox ACD



Exponential ACD



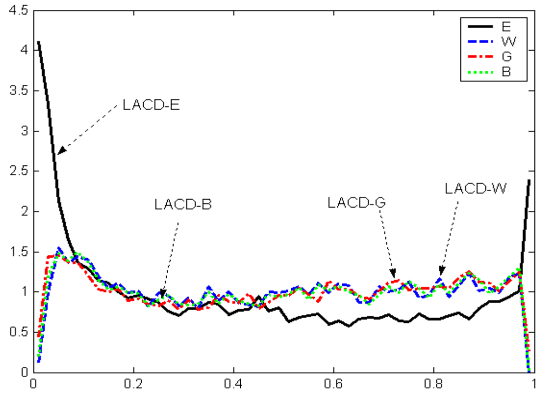
Threshold ACD



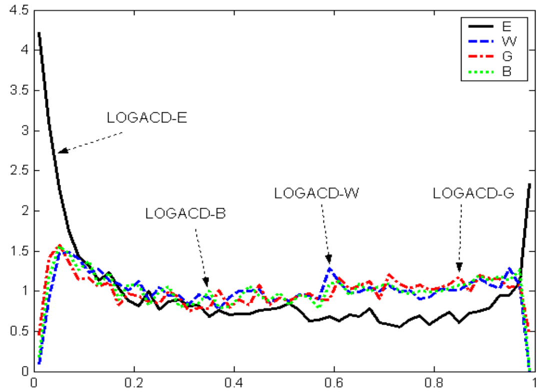
Regime switching ACD

Note: LACD, LOGACD, BCACD, EXACD, TACD and RSACD represent linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models respectively. “ E, W,G ”and “ B ”denote the ACD models based on standard exponential, Weibull, Generalized Gamma and Burr innovation respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for out-of-sample forecasting.

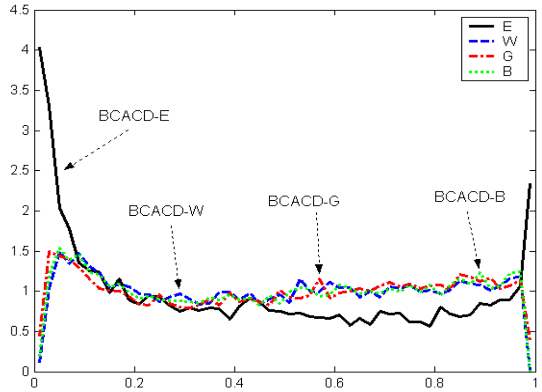
Figure 9-2. Histogram of in-sample generalized residuals of Yen/Dollar



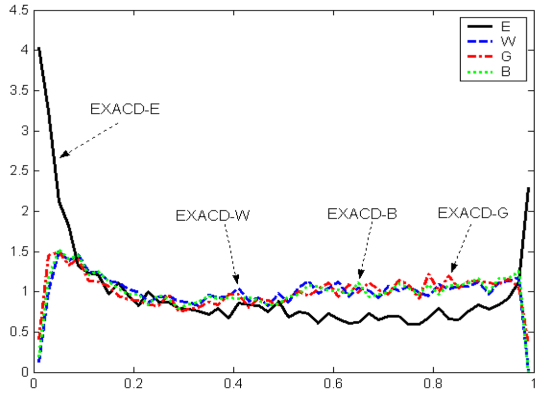
Linear ACD



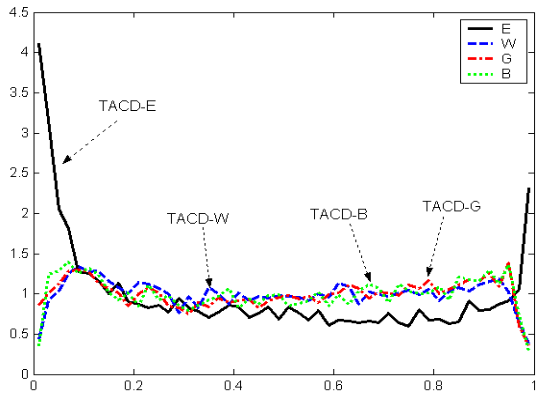
Log ACD



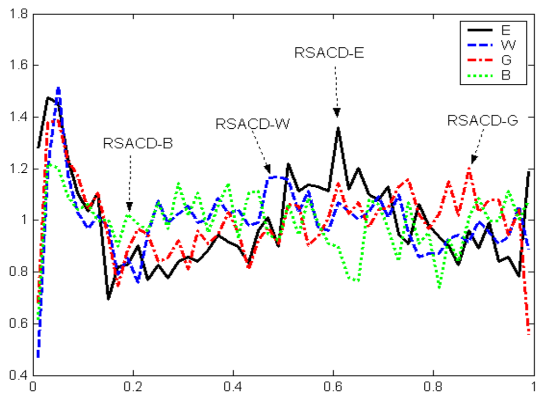
Box-Cox ACD



Exponential ACD



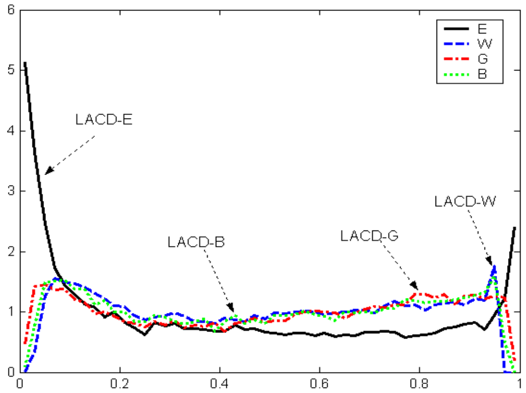
Threshold ACD



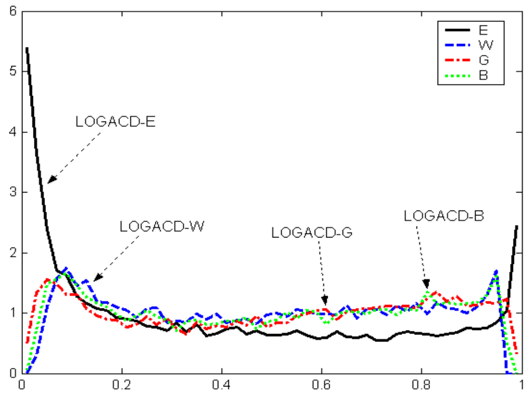
Regime switching ACD

Note: LACD, LOGACD, BCACD, EXACD, TACD and RSACD represent linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models respectively. “ E, W,G ”and “ B ”denote the ACD models based on standard exponential, Weibull, Generalized Gamma and Burr innovation respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for out-of-sample forecasting.

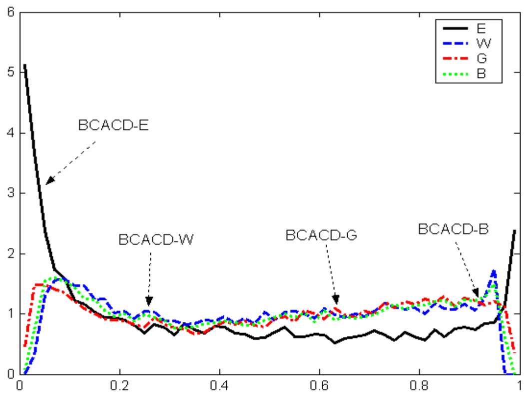
Figure 10-1: Histogram of out-of-sample forecasted generalized residuals of Euro/Dollar



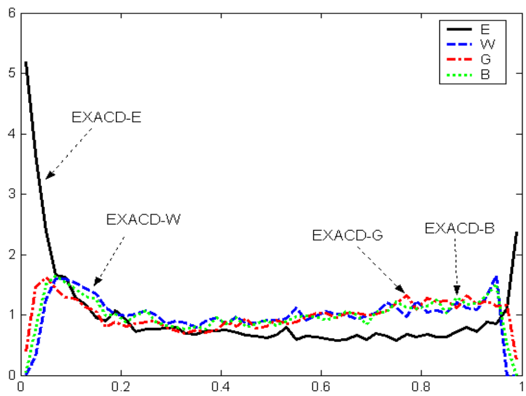
Linear ACD



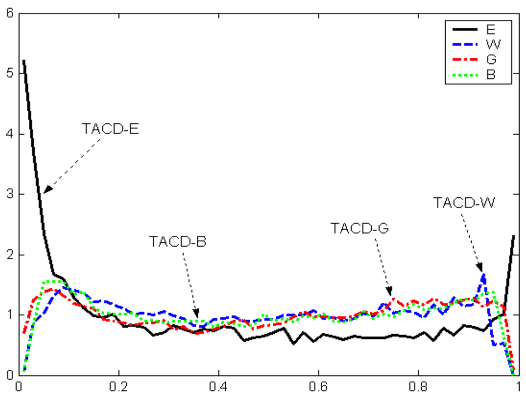
Log ACD



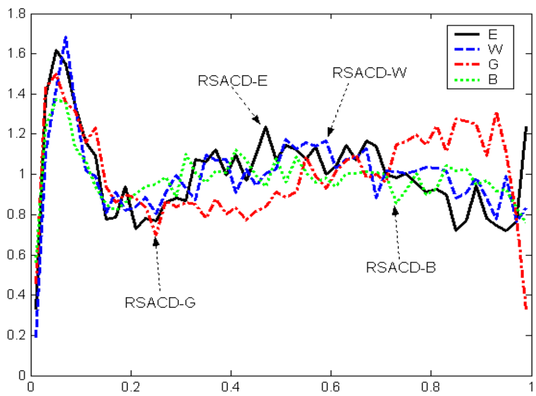
Box-Cox ACD



Exponential ACD



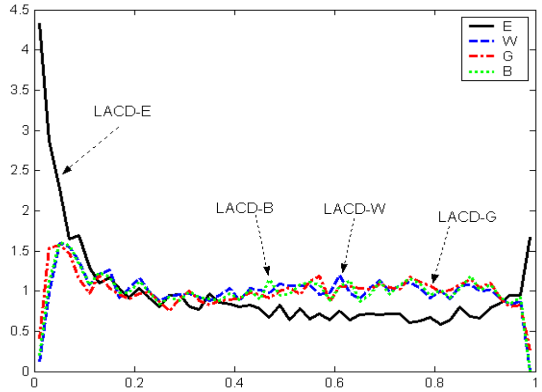
Threshold ACD



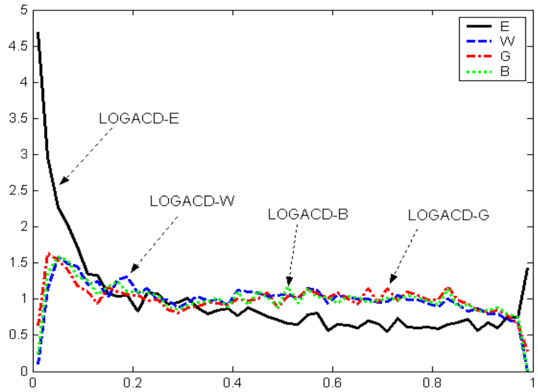
Regime switching ACD

Note: LACD, LOGACD, BCACD, EXACD, TACD and RSACD represent linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models respectively. “ E, W,G ”and “ B ”denote the ACD models based on standard exponential, Weibull, Generalized Gamma and Burr innovation respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for out-of-sample forecasting.

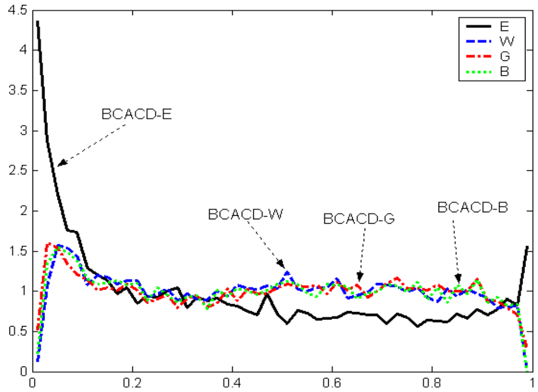
Figure 10-2: Histogram of out-of-sample forecasted generalized residuals of Yen/Dollar



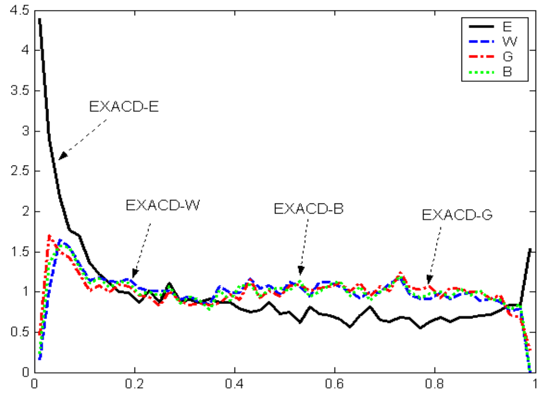
Linear ACD



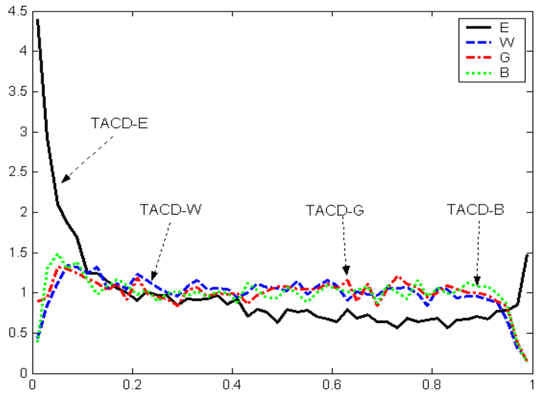
Log ACD



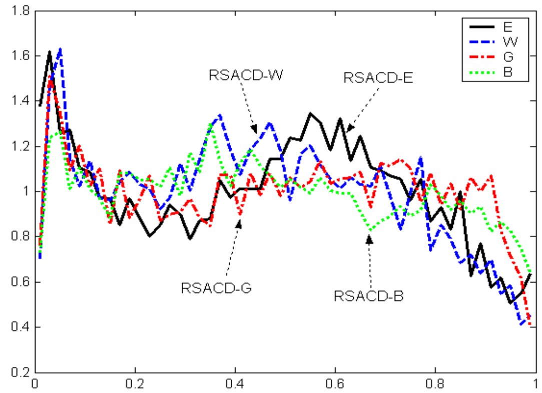
Box-Cox ACD



Exponential ACD



Threshold ACD



Regime switching ACD

Note: LACD, LOGACD, BCACD, EXACD, TACD and RSACD represent linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models respectively. “ E, W,G ”and “ B ”denote the ACD models based on standard exponential, Weibull, Generalized Gamma and Burr innovation respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for out-of-sample forecasting.

Table 1: Summary statistics for price durations of Euro/Dollar and Yen/Dollar

	Euro/Dollar		Yen/Dollar	
	Raw Duration	Deseasonalized Duration	Raw Duration	Deseasonalized Duration
the whole sample				
Sample size	20584	20584	15818	15818
Mean	213.89	1	276.31	1
Std. Dev.	500.64	1.7068	570.63	1.6293
Minimum	1	0.0009	1	0.0012
Median	57	0.3427	99	0.4166
Maximum	12666	32.363	12697	26.354
First half sample				
Sample size	10292	10292	7909	7909
Mean	217.69	1.02	331.51	1.20
Std. Dev.	542.92	1.77	896.96	1.9337
Minimum	1	0.001	1	0.001
Median	48	0.3094	115	0.4966
Maximum	12666	21.862	12697	26.354
Second half sample				
Sample size	10292	10292	7909	7909
Mean	210.1	0.9816	221.11	0.8013
Std. Dev.	454.43	1.6459	399.28	1.2212
Minimum	1	0.0009	1	0.0014
Median	64	0.3725	87	0.3592
Maximum	9654	32.363	9191	19.403

Raw price durations are measured in seconds by the time interval between two bid-ask quotes during which a cumulative change in the mid-price of at least 0.0005 for Euro/Dollar and 0.05 for Yen/Dollar is observed. The quotes obtained from Olsen & Associates are intra-day Euro/Dollar and Yen/Dolla exchange rates from July 1, 2000 to June 30, 2001 on Wednesdays. Deseasonalized price durations are produced from raw price durations by filtering out the time-of-day effects.

Table 2: Parameter estimates of ACD models for Euro/Dollar and Yen/Dollar price durations

This table reports maximum likelihood estimates of linear, logarithmic, Box-Cox, Exponential, Threshold, and Markov Switching models for Euro/Dollar and Yen/Dollar exchange rate price durations. For each model, four commonly used innovation distributions—exponential, Weibull, generalized Gamma and Burr distributions denoted by "E, W, G" and "B" respectively are considered. The whole sample are deseasonalized price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation. The numbers in the parentheses are standard errors for the estimates. The density function for innovation distribution are as follows:

$$f(\varepsilon_i) = \begin{cases} \exp(-\varepsilon_i), & \text{if } \varepsilon_i \sim E, \\ \gamma \left[\Gamma\left(1+\frac{1}{\gamma}\right)\right]^{\gamma} \varepsilon_i^{\gamma-1} \exp\left\{-\left[\Gamma\left(1+\frac{1}{\gamma}\right)\varepsilon_i\right]^{\gamma}\right\},\ \gamma>0, & \text{if } \varepsilon_i \sim W(\gamma), \\ \frac{\gamma \varepsilon_i^{\gamma\lambda-1}}{\Gamma(\lambda)} \left[\frac{\Gamma(\lambda+1/\gamma)}{\Gamma(\lambda)}\right]^{\gamma\lambda} \exp\left\{-\left[\frac{\varepsilon_i \Gamma(\lambda+1/\gamma)}{\Gamma(\lambda)}\right]^{\gamma}\right\},\ \lambda,\gamma>0, & \text{if } \varepsilon_i \sim G(\gamma,\lambda), \\ \frac{\gamma}{c}\left(\frac{\varepsilon_i}{c}\right)^{\gamma-1}\left[1+\lambda\left(\frac{\varepsilon_i}{c}\right)^{\gamma}\right]^{-(1+\lambda^{-1})}, c=\frac{(\lambda)^{1+\gamma^{-1}}\Gamma(1+\lambda^{-1})}{\Gamma(1+\gamma^{-1})\Gamma(\lambda^{-1}-\gamma^{-1})},\ \gamma>\lambda>0, & \text{if } \varepsilon_i \sim B(\gamma,\lambda). \end{cases}$$

A. Linear ACD model (LINACD)

	Euro/Dollar				Yen/Dollar			
Parameter	E	W	G	B	E	W	G	B
ω	0.0418	0.0562	0.088	0.0653	0.0521	0.0616	0.0792	0.0668
	(0.0044)	(0.0072)	(0.0102)	(0.008)	(0.0058)	(0.0098)	(0.0129)	(0.0109)
α	0.1699	0.2551	0.4096	0.3482	0.1925	0.2466	0.3167	0.2767
	(0.0091)	(0.0174)	(0.0281)	(0.0254)	(0.0098)	(0.0177)	(0.026)	(0.0221)
β	0.7997	0.7139	0.6059	0.656	0.7767	0.7244	0.6634	0.6997
	(0.0115)	(0.019)	(0.0227)	(0.0207)	(0.0116)	(0.0202)	(0.0266)	(0.0233)
γ		0.6239	0.2489	0.6968		0.6799	0.471	0.7095
		(0.0047)	(0.0186)	(0.01)		(0.0059)	(0.0261)	(0.0105)
λ			5.2979	0.1963			1.8703	0.0733
			(0.7491)	(0.0247)			(0.1795)	(0.0218)
Log-likelihood	-8943.24	-6452.94	-6292.28	-6407.41	-8310.07	-7116.01	-7087.59	-7109.39

The specified linear ACD model is
$$\begin{cases} X_i = \psi_i \varepsilon_i, \\ \psi_i = \omega + \alpha X_{i-1} + \beta \psi_{i-1}, \\ \varepsilon_i \sim i.i.d.\ E \text{ or } W(\gamma) \text{ or } G(\gamma,\lambda) \text{ or } B(\gamma,\lambda). \end{cases}$$

B. Log ACD model (LOGACD)

	Euro/Dollar				Yen/Dollar			
Parameter	E	W	G	B	E	W	G	B
ω	0.1334	0.1712	0.2652	0.2379	0.1294	0.1652	0.2244	0.1989
	(0.0057)	(0.0099)	(0.0159)	(0.0158)	(0.0067)	(0.0117)	(0.0166)	(0.0155)
α	0.115	0.1512	0.1983	0.1876	0.1105	0.1436	0.184	0.1679
	(0.0046)	(0.0079)	(0.0093)	(0.0095)	(0.0052)	(0.0091)	(0.0112)	(0.011)
β	0.7749	0.7266	0.6596	0.6823	0.7812	0.7215	0.6485	0.6795
	(0.0118)	(0.0184)	(0.0201)	(0.0196)	(0.0137)	(0.0238)	(0.0275)	(0.0268)
γ		0.6208	0.2207	0.706		0.6748	0.4138	0.722
		(0.0047)	(0.0181)	(0.0104)		(0.0059)	(0.0245)	(0.0111)
λ			6.6539	0.23			2.3356	0.1171
			(1.0456)	(0.026)			(0.2456)	(0.024)
Log-likelihood	-9050.77	-6480.52	-6289.94	-6422.23	-8401.26	-7145.65	-7098.11	-7130.44

The specified log ACD model is $\begin{cases} X_i = \psi_i \varepsilon_i, \\ \ln \psi_i = \omega + \alpha \ln X_{i-1} + \beta \ln \psi_{i-1}, \\ \varepsilon_i \sim i.i.d.\ E \text{ or } W(\gamma) \text{ or } G(\gamma, \lambda) \text{ or } B(\gamma, \lambda). \end{cases}$

C: Box-Cox ACD model (BCACD)

	Euro/Dollar				Yen/Dollar			
Parameter	E	W	G	B	E	W	G	B
ω	-0.2434	-0.4107	-0.66	-0.5636	-0.2611	-0.3602	-0.4891	-0.4222
	(0.0167)	(0.0388)	(0.0689)	(0.0557)	(0.0169)	(0.0343)	(0.0541)	(0.0446)
α	0.2999	0.509	0.8598	0.7251	0.3277	0.4536	0.6284	0.5372
	(0.0223)	(0.049)	(0.0831)	(0.07)	(0.0228)	(0.0446)	(0.0687)	(0.0575)
β	0.9238	0.8967	0.8659	0.8857	0.9198	0.9008	0.8764	0.8909
	(0.0067)	(0.0109)	(0.0125)	(0.0113)	(0.007)	(0.012)	(0.0148)	(0.0133)
δ	0.5569	0.4402	0.3378	0.3768	0.5604	0.491	0.4224	0.4572
	(0.0283)	(0.0358)	(0.0325)	(0.0324)	(0.031)	(0.0406)	(0.0399)	(0.0402)
γ		0.6262	0.2423	0.7045	0.6828	0.4563	0.7192	
		(0.0047)	(0.0185)	(0.0102)	(0.006)	(0.0258)	(0.0108)	
λ			5.6292	0.2089			1.9959	0.0894
			(0.8158)	(0.0249)			(0.1967)	(0.0224)
Log-likelihood	-8859.20	-6405.91	-6236.09	-6354.23	-8241.34	-7073.12	-7039.38	-7063.47

The specified Box-Cox ACD model is $\begin{cases} X_i = \psi_i \varepsilon_i, \\ \ln \psi_i = \omega + \alpha \varepsilon_{i-1}^{\delta} + \beta \ln \psi_{i-1}, \\ \varepsilon_i \sim i.i.d.\ E \text{ or } W(\gamma) \text{ or } G(\gamma, \lambda) \text{ or } B(\gamma, \lambda). \end{cases}$

D: Exponential ACD model (EXPACD)

	Euro/Dollar				Yen/Dollar			
Parameter	E	W	G	B	E	W	G	B
ω	-0.0806	-0.0876	-0.0531	-0.0681	-0.0889	-0.0965	-0.0836	-0.0964
	(0.0052)	(0.0087)	(0.0118)	(0.0107)	(0.0062)	(0.0097)	(0.0227)	(0.0103)
α	0.2061	0.292	0.4138	0.3752	0.2208	0.2773	0.3461	0.3088
	(0.0107)	(0.0068)	(0.0204)	(0.0207)	(0.01)	(0.0167)	(0.0234)	(0.0196)
β	0.9149	0.884	0.8444	0.8674	0.917	0.8956	0.8744	0.8866
	(0.0074)	(0.0092)	(0.0135)	(0.0126)	(0.0072)	(0.0122)	(0.014)	(0.0133)
δ	-0.1309	-0.2154	-0.3363	-0.2972	-0.1321	-0.183	-0.2438	-0.2114
	(0.0112)	(0.0024)	(0.0228)	(0.0225)	(0.0109)	(0.0183)	(0.0234)	(0.0211)
γ		0.6261	0.2463	0.7003		0.6829	0.4462	0.718
		(0.0047)	(0.0181)	(0.01)		(0.006)	(0.0183)	(0.0106)
λ			5.4455	0.201			2.0764	0.0861
			(0.7586)	(0.0241)			(0.1497)	(0.0219)
Log-likelihood	-8867.44	-6413.34	-6248.41	-6366.08	-8237.14	-7070.80	-7038.39	-7061.67

The specified exponential ACD model is $$\begin{cases} X_i = \psi_i \varepsilon_i, \\ \ln \psi_i = \omega + \alpha \varepsilon_{i-1} + \delta \left| \varepsilon_{i-1} - 1 \right| + \beta \ln \psi_{i-1}, \\ \varepsilon_i \sim i.i.d.\ E \text{ or } W(\gamma) \text{ or } G(\gamma, \lambda) \text{ or } B(\gamma, \lambda). \end{cases}$$

E: Threshold ACD model (TACD)

	Euro/Dollar				Yen/Dollar			
Parameter	E	W	G	B	E	W	G	B
ω_1	0.0184	0.0364	0.0478	-0.307	0.0171	0.0129	0.0187	-0.0297
	(0.0039)	(0.008)	(0.009)	(0.1024)	(0.0037)	(0.009)	(0.0106)	(0.0448)
α_1	0.2783	0.2134	0.6391	-0.0181	0.2786	0.3846	0.4425	0.0375
	(0.0207)	(0.0877)	(0.0692)	(0.0253)	(0.0191)	(0.0951)	(0.1048)	(0.0138)
β_1	0.8069	0.7376	0.6438	0.8893	0.794	0.7678	0.739	0.9158
	(0.0128)	(0.021)	(0.023)	(0.0234)	(0.0099)	(0.0179)	(0.0216)	(0.0165)
ω_2	0.4013	0.245	0.4562	0.1601	0.6003	0.2711	0.2974	0.1311
	(0.0414)	(0.0272)	(0.0582)	(0.0124)	(0.0566)	(0.0334)	(0.0375)	(0.0143)
α_2	0.0986	0.1288	0.1251	0.2227	0.0771	0.1387	0.1527	0.21
	(0.0105)	(0.0139)	(0.0206)	(0.0154)	(0.0116)	(0.0151)	(0.0177)	(0.0189)
β_2	0.6268	0.728	0.6964	0.6725	0.6059	0.7008	0.683	0.6504
	(0.0341)	(0.0276)	(0.0412)	(0.0258)	(0.04)	(0.0307)	(0.0338)	(0.0339)
γ_1		0.5734	0.2684	0.6971		0.5989	0.4867	0.7564
		(0.006)	(0.019)	(0.0092)		(0.0077)	(0.0255)	(0.0108)
γ_2		0.7001				0.7785		
		(0.0076)				(0.0091)		
λ_1			4.3446	0.3559			1.4781	0.4269
			(0.5833)	(0.0235)			(0.1352)	(0.0278)
λ_2			5.3646	0.0857			2.1655	0.0102
			(0.7043)	(0.0217)			(0.1981)	(0.0187)
Log-likelihood	-8804.91	-6304.44	-6195.43	-6292.29	-8186.21	-6950.79	-6948.84	-6944.52

The 2-regime threshold ACD model with standard exponential error is

$$X_i = \psi_i \varepsilon_i,\ \psi_i = \begin{cases} \omega_1 + \alpha_1 X_{i-1} + \beta_1 \psi_{i-1}, & \varepsilon_i \sim E \quad \text{if } X_{i-1} \leq 1.217\ (1.478) \\ \omega_2 + \alpha_2 X_{i-1} + \beta_2 \psi_{i-1}, & \varepsilon_i \sim E \quad \text{if } X_{i-1} > 1.217\ (1.478); \end{cases}$$

with Weibull error is

$$X_i = \psi_i \varepsilon_i,\ \psi_i = \begin{cases} \omega_1 + \alpha_1 X_{i-1} + \beta_1 \psi_{i-1}, & \varepsilon_i \sim W(\gamma_1) \quad \text{if } X_{i-1} \leq 0.309\ (0.377) \\ \omega_2 + \alpha_2 X_{i-1} + \beta_2 \psi_{i-1}, & \varepsilon_i \sim W(\gamma_2) \quad \text{if } X_{i-1} > 0.309\ (0.377); \end{cases}$$

with generalized gamma error is

$$X_i = \psi_i \varepsilon_i,\ \psi_i = \begin{cases} \omega_1 + \alpha_1 X_{i-1} + \beta_1 \psi_{i-1}, & \varepsilon_i \sim G(\gamma_1, \lambda_1) \quad \text{if } X_{i-1} \leq 0.755\ (0.377) \\ \omega_2 + \alpha_2 X_{i-1} + \beta_2 \psi_{i-1}, & \varepsilon_i \sim G(\gamma_1, \lambda_2) \quad \text{if } X_{i-1} > 0.755\ (0.377); \end{cases}$$

with Burr error is

$$X_i = \psi_i \varepsilon_i,\ \ln \psi_i = \begin{cases} \omega_1 + \alpha_1 \ln X_{i-1} + \beta_1 \ln \psi_{i-1}, & \varepsilon_i \sim B(\gamma_1, \lambda_1) \quad \text{if } X_{i-1} \leq 0.074\ (0.278) \\ \omega_2 + \alpha_2 \ln X_{i-1} + \beta_2 \ln \psi_{i-1}, & \varepsilon_i \sim B(\gamma_1, \lambda_2) \quad \text{if } X_{i-1} > 0.074\ (0.278). \end{cases}$$

Estimation is performed by a grid search over threshold values r and by maximizing the likelihood function conditional on given r. The first threshold values are for Euro/Dollar and threshold values in parentheses are for Yen/Dollar.

F: Regime-switching ACD model (RSACD)

	Euro/Dollar				Yen/Dollar			
Parameter	E	W	G	B	E	W	G	B
p_{11}	0.6116	0.5752	0.6558	0.714	0.4443	0.614	0.6123	0.6867
	(0.0129)	(0.0244)	(0.0403)	(0.012)	(0.0161)	(0.0333)	(0.0409)	(0.0165)
p_{22}	0.7514	0.86	0.3544	0.7552	0.5827	0.9008	0.3528	0.8493
	(0.0096)	(0.0086)	(0.038)	(0.01)	(0.0285)	(0.0084)	(0.0285)	(0.0095)
ω_1	-2.4913	-3.1667	0.2437	0.1061	-1.1389	-2.5765	0.2345	0.0821
	(0.0594)	(0.0876)	(0.0347)	(0.0502)	(0.0828)	(0.1175)	(0.0315)	(0.0637)
α_1	0.0348	-0.0367	0.0165	0.0176	0.4859	0.0881	-0.0004	0.0119
	(0.0176)	(0.0212)	(0.0119)	(0.0108)	(0.0296)	(0.0342)	(0.0099)	(0.0135)
β_1	0.5091	0.2541	0.9164	0.909	0.5476	0.0346	0.9702	0.9733
	(0.0623)	(0.0632)	(0.0234)	(0.0253)	(0.0812)	(0.0891)	(0.0221)	(0.0333)
ω_2	0.5118	0.3491	-0.297	0.0596	0.5344	0.3656	-0.2973	-0.1517
	(0.0123)	(0.0138)	(0.0726)	(0.0241)	(0.021)	(0.0192)	(0.0586)	(0.0327)
α_2	0.0498	0.0792	0.663	0.0601	0.0623	0.0911	0.6979	0.0765
	(0.0072)	(0.0085)	(0.034)	(0.0076)	(0.0095)	(0.0095)	(0.0257)	(0.0078)
β_2	0.5343	0.5789	0.2824	0.9226	0.8168	0.4611	0.0796	0.8949
	(0.0334)	(0.0362)	(0.0677)	(0.0197)	(0.0294)	(0.048)	(0.0654)	(0.0172)
γ_1		1.1374	0.4534	1.3211		1.0298	0.675	1.186
		(0.036)	(0.0306)	(0.0276)		(0.0443)	(0.0351)	(0.0308)
γ_2		0.7616				0.8076		
		(0.0105)				(0.0126)		
λ_1			1.9161	1.2917			1.2412	0.9874
			(0.2078)	(0.028)			(0.1212)	(0.0561)
λ_2			2.864	0.4644			1.4539	0.3593
			(0.4258)	(0.0393)			(0.1248)	(0.0365)
Log-likelihood	-6181.97	-6027.42	-6123.99	-5797.49	-7001.39	-6935.41	-6864.71	-6761.33

The 2-regime switching ACD model is $\begin{cases} X_i = \psi_i \varepsilon_i, \\ \ln \psi_i^{(S_i)} = \omega(S_i) + \alpha(S_i) \ln X_{i-1} + \beta(S_i) \ln \psi_{i-1},\ S_i = 1, 2 \\ \varepsilon_i^{(S_i)} = X_i / \psi_i^{(S_i)} \sim E \text{ or } W(\gamma_{s_t}) \text{ or } G(\gamma_1, \lambda_{s_t}) \text{ or } B(\gamma_1, \lambda_{s_t}), \end{cases}$
with constant transition probability $p_{11} = P(S_i = 1 | S_{i-1} = 1), p_{22} = P(S_i = 2 | S_{i-1} = 2)$.

Table 3: Nonparametric portmanteau density evaluation statistics for in-sample and out-of-sample performance of ACD models

Panel A. In-sample performance

	Euro/Dollar			Yen/Dollar		
Model	W(5)	W(10)	W(20)	W(5)	W(10)	W(20)
LINACD-E	3813.51	5081.27	6923.74	1495.65	1960.61	2663.42
LINACD-W	278.63	372.83	503.40	86.24	103.31	130.88
LINACD-G	147.68	191.76	251.17	80.78	96.29	121.90
LINACD-B	224.73	299.53	402.06	86.20	103.52	131.20
LOGACD-E	3933.91	5309.79	7322.43	1555.12	2051.11	2812.81
LOGACD-W	319.33	425.80	577.57	92.81	109.19	139.41
LOGACD-G	155.43	198.33	258.70	75.24	87.45	109.84
LOGACD-B	252.51	333.36	449.23	87.97	103.09	130.75
BCACD-E	3638.72	5037.01	6930.56	1428.47	1883.91	2573.63
BCACD-W	287.89	395.77	547.85	81.79	97.66	124.98
BCACD-G	145.29	187.74	245.00	73.95	88.11	113.30
BCACD-B	234.31	318.05	440.41	80.69	96.52	123.89
EXPACD-E	3734.41	5004.18	6849.53	1432.69	1889.77	2581.31
EXPACD-W	287.13	385.01	522.15	84.36	100.75	129.74
EXPACD-G	157.18	203.58	267.08	78.90	94.26	121.42
EXPACD-B	235.82	314.37	423.49	84.47	100.98	130.09
TACD-E	3643.36	4860.84	6645.82	1259.82	1744.29	2427.03
TACD-W	198.24	264.49	356.30	59.35	72.65	95.88
TACD-G	136.93	177.77	233.75	57.79	70.61	93.56
TACD-B	202.55	271.68	368.56	62.63	80.92	107.41
RSACD-E	88.07	113.73	150.30	84.27	112.54	156.54
RSACD-W	63.78	82.47	108.33	26.14	33.27	42.13
RSACD-G	113.60	149.06	196.24	35.38	44.09	58.92
RSACD-B	29.47	36.34	44.78	16.99	20.84	27.15

The table reports the evaluation statistics $W(p)$ for the in-sample density forecasting performance of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr error distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation. The $W(p)$ statistics are asymptotically one sided $N(0,1)$ distribution and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Panel B. Out-of-sample performance

	Euro/Dollar			Yen/Dollar		
Model	W(5)	W(10)	W(20)	W(5)	W(10)	W(20)
LINACD-E	3049.10	4039.90	5567.52	1447.52	1901.04	2539.42
LINACD-W	235.98	312.96	425.11	96.72	117.18	150.61
LINACD-G	169.83	221.57	298.76	83.75	99.38	125.43
LINACD-B	209.22	275.83	375.10	90.71	108.60	138.25
LOGACD-E	3145.42	4211.61	5851.44	1708.47	2276.17	3089.67
LOGACD-W	255.00	332.78	451.05	133.64	168.80	224.70
LOGACD-G	165.90	206.67	273.05	97.95	120.68	157.02
LOGACD-B	212.47	271.68	365.42	112.32	140.37	184.64
BCACD-E	2998.12	3984.67	5509.08	1433.75	1895.51	2555.39
BCACD-W	231.68	305.99	416.97	102.11	126.22	164.81
BCACD-G	163.13	210.12	281.21	84.58	103.32	132.53
BCACD-B	201.23	262.46	355.50	92.46	113.38	146.07
EXPACD-E	3025.14	4021.55	5556.89	1451.46	1918.08	2586.03
EXPACD-W	239.88	316.55	431.20	105.70	130.70	171.59
EXPACD-G	175.90	227.04	305.39	90.55	110.11	142.17
EXPACD-B	212.56	277.35	375.33	97.06	118.44	153.82
TACD-E	2995.36	3973.34	5485.29	1294.17	1775.72	2499.07
TACD-W	176.27	229.52	312.14	102.59	135.81	185.91
TACD-G	157.86	204.62	274.33	78.05	99.67	133.66
TACD-B	183.80	241.43	327.71	83.63	107.79	145.58
RSACD-E	121.13	156.22	207.69	106.29	144.92	201.26
RSACD-W	81.16	103.70	136.47	146.46	200.07	276.79
RSACD-G	132.12	175.02	239.88	52.08	67.85	91.41
RSACD-B	33.47	39.68	49.50	34.58	42.49	54.61

The table reports the evaluation statistics $W(p)$ for the out-of-sample density forecasting performance of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr error distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. The $W(p)$ statistics are asymptotically one sided $N(0,1)$ distribution and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Table 4: Separate Diagnostic Statistics for in-sample and out-of-sample generalized residuals of ACD models

Panel A. In-sample generalized residuals of Euro/Dollar

Model	$M_z(0,0)$	$M_z(1,0)$	$M_z(2,0)$	$M_z(3,0)$	$M_z(4,0)$	$M_z(1,1)$	$M_z(1,2)$	$M_z(2,1)$	$M_z(2,2)$	$M_z(3,3)$	$M_z(4,4)$
LINACD-E	112.20	111.90	90.56	103.50	85.35	111.80	39.30	90.50	43.43	96.57	41.10
LINACD-W	70.50	70.18	35.69	34.96	23.74	70.04	16.43	35.71	12.57	24.65	12.92
LINACD-G	29.73	29.42	34.47	9.85	25.93	29.30	16.77	34.52	11.12	5.59	13.75
LINACD-B	40.14	39.85	37.62	18.59	27.49	39.74	17.69	37.66	12.47	12.47	14.51
LOGACD-E	105.40	105.00	84.87	88.63	76.11	105.00	1.59	84.74	52.30	76.74	47.66
LOGACD-W	55.25	54.93	23.12	22.08	13.93	54.81	24.08	23.04	26.60	15.42	27.69
LOGACD-G	19.96	19.70	23.21	10.56	18.13	19.57	30.98	23.14	25.60	6.97	28.01
LOGACD-B	26.23	25.98	24.93	12.98	17.63	25.86	28.96	24.86	26.67	8.78	28.83
BCACD-E	92.35	92.02	90.25	82.31	84.21	91.94	22.37	90.19	43.28	75.31	41.16
BCACD-W	49.68	49.36	34.40	21.81	23.22	49.28	5.73	34.40	15.81	14.81	16.32
BCACD-G	16.80	16.54	32.70	9.52	25.94	16.49	5.51	32.70	15.86	6.42	18.13
BCACD-B	23.63	23.36	35.85	13.44	26.83	23.32	5.17	35.86	16.40	8.97	18.32
EXPACD-E	92.18	91.86	88.96	82.88	82.66	91.83	17.83	88.87	45.49	80.34	43.35
EXPACD-W	54.79	54.48	32.20	23.73	21.02	54.44	5.89	32.18	17.54	19.48	18.10
EXPACD-G	21.65	21.36	30.66	7.11	23.60	21.34	6.21	30.65	16.23	4.30	18.92
EXPACD-B	27.95	27.67	34.01	12.39	24.51	27.66	5.91	34.00	17.54	9.19	19.89
TACD-E	96.15	95.81	91.54	86.40	85.73	95.73	27.35	91.48	43.34	82.16	42.44
TACD-W	22.99	22.84	6.13	9.66	22.75	22.81	8.22	6.05	19.72	7.80	20.07
TACD-G	13.79	13.62	6.31	6.40	2.40	13.60	5.39	6.24	24.89	3.38	25.99
TACD-B	8.18	8.07	7.97	6.25	1.98	8.02	19.77	7.94	14.80	2.67	16.20
RSACD-E	44.25	44.23	3.27	33.24	4.95	44.14	45.14	3.18	20.04	33.82	20.18
RSACD-W	47.48	47.42	9.41	32.60	7.51	47.29	56.41	9.30	19.47	32.27	19.31
RSACD-G	9.88	9.73	22.73	6.50	30.15	9.62	28.13	22.72	18.59	4.89	21.15
RSACD-B	13.60	13.57	1.53	8.83	3.73	13.39	43.86	1.41	27.54	8.44	29.97

This table reports $M_z(m,l)$ diagnostic test statistics for the serial dependence of generalized residuals $\{Z_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_z(0,0)$ represents statistics on i.i.d. test. $M_z(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_z(l,1), M_z(2,2), M_z(3,3), M_z(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_z(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Panel B. In-sample generalized residuals of Yen/Dollar

Model	$M_z(0,0)$	$M_z(1,0)$	$M_z(2,0)$	$M_z(3,0)$	$M_z(4,0)$	$M_z(1,1)$	$M_z(1,2)$	$M_z(2,1)$	$M_z(2,2)$	$M_z(3,3)$	$M_z(4,4)$
LINACD-E	78.69	78.43	93.45	84.42	94.01	78.27	38.52	93.41	37.04	77.48	40.84
LINACD-W	71.10	70.77	72.49	52.68	62.99	70.60	16.43	72.48	12.74	36.05	12.15
LINACD-G	49.45	49.09	78.25	33.69	69.50	48.92	17.19	78.25	11.49	21.71	11.62
LINACD-B	60.10	59.76	75.31	43.81	66.29	59.60	16.83	75.30	12.40	29.68	12.20
LOGACD-E	89.10	88.83	88.26	89.07	83.92	88.76	7.35	88.12	57.36	79.72	61.35
LOGACD-W	63.84	63.49	62.21	38.35	50.83	63.38	10.80	62.06	36.59	26.64	30.74
LOGACD-G	36.67	36.28	71.33	17.23	62.05	36.18	12.14	71.18	34.11	10.37	29.25
LOGACD-B	45.71	45.34	68.08	24.70	57.60	45.24	11.74	67.93	35.72	16.18	30.55
BCACD-E	62.85	62.56	94.59	67.74	94.19	62.45	22.27	94.54	39.96	61.73	43.24
BCACD-W	52.64	52.29	76.53	35.84	64.92	52.19	4.68	76.49	17.76	24.00	14.98
BCACD-G	32.93	32.54	82.95	19.89	72.38	32.46	2.80	82.90	17.94	12.57	15.58
BCACD-B	40.99	40.63	80.27	27.06	69.21	40.54	3.81	80.23	18.18	17.89	15.64
EXPACD-E	62.06	61.77	94.34	67.04	94.31	61.71	17.59	94.28	43.15	65.28	46.77
EXPACD-W	53.48	53.13	77.36	36.63	65.78	53.07	4.23	77.29	20.41	28.28	17.19
EXPACD-G	36.97	36.58	83.77	22.93	73.04	36.53	4.05	83.70	19.57	17.44	17.17
EXPACD-B	42.99	42.63	81.14	28.62	69.96	42.58	3.99	81.07	20.41	22.30	17.61
TACD-E	65.46	65.20	92.91	72.84	91.34	65.11	22.64	92.87	41.12	70.19	45.50
TACD-W	22.28	22.24	1.93	15.15	2.00	22.21	3.38	1.91	10.90	12.43	9.20
TACD-G	24.45	24.36	6.43	13.90	0.39	24.32	3.19	6.42	12.39	11.46	10.13
TACD-B	10.36	10.23	30.49	6.93	12.47	10.23	1.57	30.47	12.57	6.81	10.83
RSACD-E	18.06	17.95	13.62	9.41	14.32	17.88	8.32	13.63	11.19	6.35	10.24
RSACD-W	40.37	40.34	12.00	32.88	9.09	40.27	28.28	11.92	14.64	30.20	13.00
RSACD-G	15.61	15.55	7.59	9.04	10.91	15.51	3.94	7.56	16.52	6.63	18.62
RSACD-B	4.27	4.23	10.74	4.29	9.57	4.15	21.67	10.70	10.63	5.38	11.41

This table reports $M_z(m,l)$ diagnostic test statistics for the serial dependence of generalized residuals $\{Z_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_z(0,0)$ represents statistics on i.i.d. test. $M_z(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_z(l,1), M_z(2,2), M_z(3,3), M_z(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_z(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Panel C. Out-of-sample generalized residuals of Euro/Dollar

Model	M(0,0)	M(1,0)	M(2,0)	M(3,0)	M(4,0)	M(1,1)	M(1,2)	M(2,1)	M(2,2)	M(3,3)	M(4,4)
LINACD-E	47.59	47.19	123.30	55.82	119.30	47.09	34.36	123.20	56.38	56.08	55.38
LINACD-W	38.87	38.48	93.48	25.56	75.59	38.38	22.02	93.47	10.92	21.73	13.62
LINACD-G	20.60	20.21	91.52	14.21	78.06	20.11	24.31	91.52	6.90	13.89	12.75
LINACD-B	23.28	22.92	94.11	18.01	78.68	22.83	23.49	94.10	9.19	17.00	13.97
LOGACD-E	46.96	46.55	120.40	48.00	115.40	46.51	11.42	120.20	81.18	44.34	77.00
LOGACD-W	32.46	32.11	83.40	24.10	65.12	32.03	20.25	83.24	37.34	20.70	35.70
LOGACD-G	18.36	18.06	81.77	27.01	68.91	17.98	23.15	81.62	30.31	23.60	33.39
LOGACD-B	20.13	19.84	83.88	25.91	68.27	19.76	23.41	83.73	34.13	23.17	34.90
BCACD-E	35.58	35.15	125.10	41.99	121.30	35.10	20.40	125.00	63.99	42.95	62.02
BCACD-W	27.26	26.88	97.34	21.51	78.30	26.84	8.28	97.28	18.64	19.22	20.37
BCACD-G	14.76	14.41	94.42	22.01	80.60	14.40	5.84	94.36	15.51	20.49	20.90
BCACD-B	16.44	16.11	97.58	22.13	81.23	16.10	6.45	97.52	17.62	20.63	21.35
EXPACD-E	35.85	35.41	125.20	41.62	121.30	35.41	16.03	125.10	67.53	45.87	65.03
EXPACD-W	30.10	29.71	97.05	21.10	77.96	29.70	8.01	96.95	20.84	20.31	21.91
EXPACD-G	17.05	16.68	95.07	17.09	80.73	16.67	8.60	94.97	14.85	14.55	20.30
EXPACD-B	18.04	17.68	98.42	18.88	81.45	17.68	7.80	98.33	18.64	16.99	22.19
TACD-E	38.61	38.17	127.10	43.62	124.80	38.13	23.66	127.10	65.88	46.97	66.24
TACD-W	12.57	12.51	12.56	12.94	5.62	12.50	9.56	12.54	14.78	12.83	13.80
TACD-G	12.68	12.49	50.14	18.23	31.63	12.47	10.00	50.09	22.31	15.30	27.02
TACD-B	10.30	10.15	52.08	16.25	30.64	10.13	11.66	52.03	12.84	13.85	13.18
RSACD-E	40.79	40.67	35.69	34.25	18.62	40.63	30.35	35.57	28.98	36.25	25.63
RSACD-W	42.37	42.20	57.36	35.76	50.93	42.15	36.79	57.21	31.97	35.81	33.61
RSACD-G	13.52	13.47	11.22	16.46	11.21	13.44	17.43	11.14	23.29	18.49	26.80
RSACD-B	24.63	24.45	36.86	15.14	30.51	24.35	24.94	36.82	25.49	16.20	27.29

This table reports $M_z(m,l)$ diagnostic test statistics for the serial dependence of generalized residuals $\{Z_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_z(0,0)$ represents statistics on i.i.d. test. $M_z(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_z(l,1), M_z(2,2), M_z(3,3), M_z(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_z(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Panel D. Out-of-sample generalized residuals of Yen/Dollar

Model	M(0,0)	M(1,0)	M(2,0)	M(3,0)	M(4,0)	M(1,1)	M(1,2)	M(2,1)	M(2,2)	M(3,3)	M(4,4)
LINACD-E	70.29	70.01	105.10	81.70	101.00	69.90	40.51	105.00	55.51	83.87	57.26
LINACD-W	67.90	67.57	79.29	52.02	65.41	67.42	22.56	79.26	20.49	40.35	16.10
LINACD-G	47.79	47.42	83.62	35.30	70.55	47.27	23.81	83.61	17.84	25.97	15.06
LINACD-B	57.23	56.88	80.93	43.80	67.60	56.74	22.86	80.91	19.45	33.52	15.74
LOGACD-E	72.53	72.27	98.09	83.60	93.81	72.27	8.72	97.96	72.34	78.73	72.14
LOGACD-W	51.80	51.46	74.76	37.43	59.54	51.38	10.24	74.61	46.02	26.15	33.74
LOGACD-G	27.28	26.91	80.94	18.43	66.06	26.82	13.10	80.82	42.04	11.00	31.21
LOGACD-B	35.11	34.75	78.22	24.86	63.35	34.66	12.33	78.10	44.05	16.10	32.89
BCACD-E	53.74	53.45	100.20	63.77	96.56	53.40	22.21	100.20	59.44	66.05	61.59
BCACD-W	47.83	47.49	81.17	37.00	66.98	47.42	7.77	81.11	28.37	28.87	21.39
BCACD-G	29.40	29.02	85.59	24.12	71.83	28.96	5.22	85.54	26.73	17.01	20.70
BCACD-B	36.66	36.31	83.17	29.76	69.51	36.25	6.36	83.11	27.74	22.36	21.31
EXPACD-E	53.24	52.94	100.50	63.67	97.11	52.95	18.52	100.40	62.59	71.57	64.80
EXPACD-W	49.53	49.18	83.32	38.55	68.90	49.15	8.28	83.24	30.36	35.55	23.36
EXPACD-G	34.13	33.75	87.76	27.26	73.80	33.72	8.03	87.68	27.47	24.38	22.06
EXPACD-B	39.42	39.06	85.53	31.76	71.54	39.04	7.75	85.45	29.12	29.14	22.91
TACD-E	41.06	40.74	97.96	51.75	95.79	40.71	24.22	97.88	64.03	58.31	66.80
TACD-W	10.44	10.43	2.51	12.23	-0.37	10.43	5.39	2.47	15.83	9.86	10.43
TACD-G	12.63	12.58	7.97	13.13	-0.25	12.57	5.70	7.94	17.34	10.11	11.17
TACD-B	3.91	3.78	35.36	5.52	15.63	3.78	4.26	35.33	16.55	4.95	10.91
RSACD-E	7.95	7.87	9.85	5.03	9.52	7.82	13.15	9.84	12.36	4.97	10.62
RSACD-W	13.30	13.21	29.13	9.99	19.67	13.14	30.65	29.09	19.63	11.18	14.01
RSACD-G	6.37	6.32	6.76	5.07	8.62	6.29	8.76	6.71	21.71	5.05	25.20
RSACD-B	4.55	4.48	13.65	2.58	11.81	4.37	31.14	13.62	12.28	4.04	9.87

This table reports $M_z(m,l)$ diagnostic test statistics for the serial dependence of generalized residuals $\{Z_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_z(0,0)$ represents statistics on i.i.d. test. $M_z(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_z(l,1), M_z(2,2), M_z(3,3), M_z(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_z(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Table 5: Separate Inference Statistics for in-sample and out-of-sample residuals of ACD models

Panel A. In-sample residuals of Euro/Dollar

Model	$M_\varepsilon(0,0)$	$M_\varepsilon(1,0)$	$M_\varepsilon(2,0)$	$M_\varepsilon(3,0)$	$M_\varepsilon(4,0)$	$M_\varepsilon(1,1)$	$M_\varepsilon(1,2)$	$M_\varepsilon(2,1)$	$M_\varepsilon(2,2)$	$M_\varepsilon(3,3)$	$M_\varepsilon(4,4)$
LINACD-E	40.56	11.05	1.43	-0.79	-1.28	6.56	-0.43	0.78	-1.25	-0.79	-0.85
LINACD-W	22.07	7.46	3.54	-0.37	-1.12	7.57	0.06	2.27	-1.50	-1.19	-0.73
LINACD-G	22.04	16.73	6.13	0.20	-0.90	15.43	1.20	3.93	-1.01	-1.61	-1.15
LINACD-B	21.19	14.10	4.88	-0.12	-1.00	13.15	0.56	3.04	-1.39	-1.46	-0.98
LOGACD-E	61.42	19.94	1.02	-0.08	-0.80	26.59	13.95	0.50	5.10	-0.53	-1.41
LOGACD-W	40.18	8.44	3.06	0.74	-0.38	10.73	8.34	0.06	4.01	0.44	-1.75
LOGACD-G	24.73	6.59	5.81	1.90	0.38	2.26	4.09	1.41	5.42	6.46	2.71
LOGACD-B	27.92	6.93	5.14	1.52	0.11	3.46	4.99	1.08	5.24	4.60	0.65
BCACD-E	35.44	7.61	1.97	-0.46	-1.12	4.87	-0.50	1.02	-1.45	-1.12	-0.71
BCACD-W	21.25	5.64	4.19	0.19	-0.76	4.36	-0.28	2.39	-0.84	-1.68	-1.21
BCACD-G	19.41	13.24	6.55	0.98	-0.26	8.23	0.37	4.43	2.58	-0.60	-1.83
BCACD-B	19.99	12.23	5.57	0.60	-0.46	7.87	0.12	3.62	0.99	-1.62	-1.61
EXPACD-E	39.75	8.36	1.56	-0.41	-1.05	4.62	-0.54	1.13	-1.49	-1.22	-0.76
EXPACD-W	24.22	3.56	3.57	0.26	-0.64	3.38	0.22	2.49	0.16	-1.81	-1.45
EXPACD-G	17.53	8.64	5.96	1.12	0.05	6.35	1.43	5.84	7.67	4.47	0.29
EXPACD-B	18.71	7.68	5.03	0.73	-0.23	5.91	1.04	4.45	4.60	0.69	-1.62
TACD-E	36.14	5.65	2.21	-0.19	-0.89	4.20	0.65	0.57	-1.17	-1.01	-0.71
TACD-W	26.86	4.47	4.65	1.13	0.13	3.58	1.64	2.78	0.57	-1.65	-1.64
TACD-G	20.10	7.15	5.12	0.67	-0.28	3.30	-0.07	3.39	0.58	-1.70	-1.60
TACD-B	25.20	4.29	2.48	-0.11	-0.79	5.99	4.85	1.74	2.74	-1.59	-1.53
RSACD-E	100.90	48.27	5.07	2.59	1.90	47.67	18.11	2.34	3.74	2.30	1.53
RSACD-W	82.47	27.87	3.40	2.08	1.01	30.17	11.52	1.59	3.78	4.12	2.57
RSACD-G	36.85	11.02	2.24	-0.36	-1.03	8.91	1.90	0.50	-0.57	-1.26	-0.85
RSACD-B	167.80	127.40	10.19	0.60	-0.97	123.90	32.04	15.11	11.72	1.28	-1.03

This table reports $M_\varepsilon(m,l)$ diagnostic test statistics for the serial dependence of standardized residuals $\{\hat{\varepsilon}_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_\varepsilon(0,0)$ represents statistics on i.i.d. test. $M_\varepsilon(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_\varepsilon(l,1), M_\varepsilon(2,2), M_\varepsilon(3,3), M_\varepsilon(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_\varepsilon(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Panel B. In-sample residuals of Yen/Dollar

Model	$M_\varepsilon(0,0)$	$M_\varepsilon(1,0)$	$M_\varepsilon(2,0)$	$M_\varepsilon(3,0)$	$M_\varepsilon(4,0)$	$M_\varepsilon(1,1)$	$M_\varepsilon(1,2)$	$M_\varepsilon(2,1)$	$M_\varepsilon(2,2)$	$M_\varepsilon(3,3)$	$M_\varepsilon(4,4)$
LINACD-E	16.41	6.77	2.19	0.58	-0.09	5.23	0.99	2.19	-0.40	-1.10	-0.98
LINACD-W	12.02	6.48	5.78	2.32	0.92	5.98	1.68	4.68	-0.15	-1.07	-0.95
LINACD-G	12.60	10.58	8.78	3.08	1.11	9.54	2.07	6.35	-0.23	-1.03	-0.85
LINACD-B	11.87	8.18	7.45	2.91	1.17	7.57	1.95	5.69	-0.13	-1.05	-0.92
LOGACD-E	42.63	22.21	3.23	1.61	1.20	32.67	31.36	2.96	8.97	0.60	-0.71
LOGACD-W	28.12	11.38	4.44	2.36	1.58	16.14	21.85	0.79	4.29	-0.35	-0.74
LOGACD-G	18.61	8.14	6.51	2.86	1.65	5.28	12.49	0.38	0.97	-0.77	-0.76
LOGACD-B	21.63	8.62	5.71	2.69	1.66	8.73	15.97	0.39	2.03	-0.65	-0.75
BCACD-E	14.12	4.70	3.50	1.77	0.88	3.91	0.31	2.24	-0.88	-1.14	-0.97
BCACD-W	10.38	5.16	7.08	3.03	1.36	3.65	-0.09	4.26	-0.81	-1.05	-0.85
BCACD-G	10.78	9.30	9.42	3.19	1.19	5.77	-0.29	5.38	-0.84	-0.98	-0.76
BCACD-B	10.33	7.37	8.64	3.26	1.32	4.86	-0.19	5.08	-0.80	-1.01	-0.80
EXPACD-E	15.54	4.73	2.89	1.65	0.91	3.57	-0.02	1.72	-0.94	-1.14	-0.97
EXPACD-W	10.54	3.56	6.11	2.96	1.45	2.65	-0.11	3.54	-0.83	-1.08	-0.88
EXPACD-G	9.36	6.05	8.66	3.48	1.41	4.33	0.13	4.89	-0.81	-1.07	-0.84
EXPACD-B	9.65	4.83	7.73	3.38	1.49	3.52	0.01	4.41	-0.81	-1.07	-0.86
TACD-E	15.46	4.41	3.89	2.54	1.55	4.23	2.92	1.34	-0.89	-1.23	-1.09
TACD-W	13.07	3.21	4.68	2.66	1.26	2.52	0.76	2.28	-1.04	-1.27	-1.08
TACD-G	10.89	3.26	6.50	3.38	1.46	1.92	0.18	3.34	-1.03	-1.16	-0.94
TACD-B	17.13	5.65	1.23	0.60	0.21	11.79	13.44	0.27	3.72	-0.08	-0.95
RSACD-E	36.89	18.44	2.05	1.12	0.91	25.70	21.56	1.61	4.82	-0.21	-0.82
RSACD-W	58.52	33.25	8.92	4.90	3.40	28.24	13.61	1.07	2.33	-0.27	-0.82
RSACD-G	16.93	6.65	3.09	1.56	0.74	6.53	2.24	1.54	-0.76	-1.24	-1.09
RSACD-B	117.50	100.80	16.23	3.17	0.44	98.16	29.62	17.85	8.41	-0.39	-1.07

This table reports $M_\varepsilon(m,l)$ diagnostic test statistics for the serial dependence of standardized residuals $\{\hat{\varepsilon}_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_\varepsilon(0,0)$ represents statistics on i.i.d. test. $M_\varepsilon(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_\varepsilon(l,1), M_\varepsilon(2,2), M_\varepsilon(3,3), M_\varepsilon(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_\varepsilon(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Panel C. Out-of-sample residuals of Euro/Dollar

Model	$M_\varepsilon(0,0)$	$M_\varepsilon(1,0)$	$M_\varepsilon(2,0)$	$M_\varepsilon(3,0)$	$M_\varepsilon(4,0)$	$M_\varepsilon(1,1)$	$M_\varepsilon(1,2)$	$M_\varepsilon(2,1)$	$M_\varepsilon(2,2)$	$M_\varepsilon(3,3)$	$M_\varepsilon(4,4)$
LINACD-E	13.45	6.54	9.38	5.15	2.72	5.20	1.16	9.06	1.34	-0.84	-0.92
LINACD-W	19.77	19.86	18.11	7.74	3.46	18.47	4.66	14.70	1.69	-0.90	-0.85
LINACD-G	36.66	39.34	21.17	6.64	2.34	32.78	6.04	15.07	0.81	-0.91	-0.77
LINACD-B	31.15	34.15	21.21	7.36	2.73	29.26	5.83	15.59	1.12	-0.92	-0.80
LOGACD-E	25.27	12.54	7.41	4.14	2.65	11.50	21.11	0.91	4.09	-0.44	-0.75
LOGACD-W	21.62	13.73	10.52	4.74	2.58	4.84	17.12	1.21	2.08	-0.84	-0.85
LOGACD-G	22.00	19.74	11.64	4.48	2.30	2.53	12.62	1.95	0.18	-1.12	-0.93
LOGACD-B	22.12	18.82	11.83	4.76	2.48	2.65	13.97	1.77	0.61	-1.12	-0.98
BCACD-E	12.24	6.36	10.20	5.07	2.56	3.91	-0.67	8.01	-0.03	-0.93	-0.85
BCACD-W	18.38	18.68	16.27	6.03	2.46	11.73	0.04	10.53	-0.24	-0.94	-0.79
BCACD-G	29.83	33.45	17.20	4.86	1.55	18.19	0.12	9.42	-0.74	-0.98	-0.74
BCACD-B	27.77	31.28	18.02	5.56	1.91	18.13	0.29	10.44	-0.53	-0.99	-0.78
EXPACD-E	12.29	5.05	8.36	4.21	2.16	2.35	-0.80	6.35	-0.24	-0.91	-0.84
EXPACD-W	15.25	13.28	13.63	5.37	2.28	7.60	-0.01	8.59	-0.34	-0.89	-0.78
EXPACD-G	24.48	26.45	16.25	5.03	1.74	14.13	0.36	8.95	-0.62	-0.86	-0.72
EXPACD-B	21.86	23.51	16.18	5.36	1.91	12.75	0.27	9.22	-0.50	-0.89	-0.74
TACD-E	11.83	4.21	8.27	2.98	0.74	1.74	-0.92	5.79	-0.71	-0.89	-0.70
TACD-W	14.42	11.14	13.07	4.72	1.51	5.45	-0.93	8.29	-0.52	-0.90	-0.75
TACD-G	21.45	22.08	15.16	4.33	1.28	9.34	-0.90	8.12	-0.89	-0.86	-0.71
TACD-B	14.29	10.98	10.82	3.67	1.41	1.02	2.07	3.42	-1.38	-0.95	-0.71
RSACD-E	67.11	47.10	10.25	3.73	2.45	50.33	24.57	8.32	1.64	-1.01	-0.71
RSACD-W	52.64	31.58	6.85	2.63	2.02	31.90	20.97	2.80	0.05	-0.78	-0.82
RSACD-G	13.37	7.97	10.16	4.94	2.42	2.83	-0.77	6.03	-1.11	-1.03	-0.84
RSACD-B	90.29	81.67	15.08	9.49	7.36	83.74	36.67	18.17	20.74	3.78	-0.90

This table reports $M_\varepsilon(m,l)$ diagnostic test statistics for the serial dependence of standardized residuals $\{\hat{\varepsilon}_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_\varepsilon(0,0)$ represents statistics on i.i.d. test. $M_\varepsilon(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_\varepsilon(l,1), M_\varepsilon(2,2), M_\varepsilon(3,3), M_\varepsilon(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_\varepsilon(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.

Panel D. Out-of-sample residuals of Yen/Dollar

Model	$M_\varepsilon(0,0)$	$M_\varepsilon(1,0)$	$M_\varepsilon(2,0)$	$M_\varepsilon(3,0)$	$M_\varepsilon(4,0)$	$M_\varepsilon(1,1)$	$M_\varepsilon(1,2)$	$M_\varepsilon(2,1)$	$M_\varepsilon(2,2)$	$M_\varepsilon(3,3)$	$M_\varepsilon(4,4)$
LINACD-E	15.47	7.02	3.71	0.84	-0.36	4.20	0.91	3.31	0.08	-1.13	-1.00
LINACD-W	10.44	5.37	6.78	2.31	0.32	4.88	2.42	6.12	0.74	-1.05	-0.98
LINACD-G	10.57	8.02	9.95	3.45	0.75	8.56	4.18	8.79	1.28	-1.05	-0.99
LINACD-B	10.08	6.42	8.37	2.97	0.64	6.53	3.21	7.45	0.99	-1.03	-0.97
LOGACD-E	40.25	23.54	2.57	0.46	-0.42	22.68	10.44	0.41	-0.51	-1.14	-0.73
LOGACD-W	24.55	10.63	3.41	0.65	-0.60	8.98	5.55	0.54	-1.30	-1.09	-0.71
LOGACD-G	15.53	6.02	6.22	1.47	-0.49	1.78	2.12	2.44	-1.52	-1.16	-0.71
LOGACD-B	18.15	6.87	4.92	0.98	-0.62	3.73	3.26	1.51	-1.50	-1.10	-0.71
BCACD-E	13.56	6.28	5.19	2.29	0.94	4.47	0.14	3.75	-0.60	-1.11	-0.93
BCACD-W	9.55	5.76	8.01	3.38	1.31	4.62	0.18	5.84	-0.45	-1.07	-0.86
BCACD-G	10.22	9.15	9.90	2.72	0.21	7.07	0.23	7.10	-0.51	-0.98	-0.70
BCACD-B	9.59	7.50	9.19	3.16	0.75	5.98	0.24	6.63	-0.48	-1.03	-0.78
EXPACD-E	14.97	7.07	4.19	1.58	0.51	4.69	-0.01	2.79	-0.87	-1.13	-0.90
EXPACD-W	9.73	4.92	6.43	2.63	1.05	3.53	-0.01	4.36	-0.75	-1.16	-0.93
EXPACD-G	8.76	6.46	8.47	3.04	0.95	4.78	0.14	5.73	-0.71	-1.15	-0.88
EXPACD-B	8.93	5.68	7.64	2.95	1.09	4.18	0.06	5.16	-0.73	-1.15	-0.90
TACD-E	12.24	7.66	6.42	2.54	1.27	5.30	0.28	3.95	-0.81	-1.06	-0.85
TACD-W	10.67	5.18	5.73	2.46	1.26	3.72	0.04	3.27	-1.18	-1.20	-0.94
TACD-G	9.40	5.37	7.02	2.97	1.48	3.67	-0.27	4.22	-1.16	-1.27	-1.01
TACD-B	14.58	5.89	1.79	0.68	0.49	6.27	2.98	0.27	-0.94	-1.15	-0.80
RSACD-E	35.67	21.31	2.31	0.68	-0.07	19.52	7.66	0.34	-0.65	-1.40	-1.02
RSACD-W	33.34	16.50	8.00	5.29	3.35	14.76	7.01	4.08	-0.29	-1.59	-1.36
RSACD-G	15.00	7.92	5.25	2.24	1.13	5.83	0.59	3.26	-0.89	-1.08	-0.86
RSACD-B	112.20	99.65	12.55	3.20	0.32	84.96	17.18	10.19	12.35	8.73	0.63

This table reports $M_\varepsilon(m,l)$ diagnostic test statistics for the serial dependence of standardized residuals $\{\hat{\varepsilon}_i\}$ of linear ACD, log ACD, Box-Cox ACD, Exponential ACD, threshold ACD and Markov regime switching models based on standard exponential, Weibull, generalized Gamma and Burr innovation distributions respectively. The whole sample are seasonally adjusted price durations from July, 1, 2000 to June 30, 2001 on Wednesdays, with total 20,584 and 15,818 observations for Euro/Dollar and Yen/Dollar respectively. The first half of the samples are used for estimation and the second half are used for forecasting. $M_\varepsilon(0,0)$ represents statistics on i.i.d. test. $M_\varepsilon(l,0), l = 1,2,3,4$ represent test statistics on martingale, ARCH effect, conditional skewness and conditional heterokurtosis respectively. And $M_\varepsilon(l,1), M_\varepsilon(2,2), M_\varepsilon(3,3), M_\varepsilon(4,4)$ are very sensitive to autocorrelations in mean, variance, skewness, and kurtosis of the generalized residuals respectively. We only show results for preliminary lag truncation order $p = 20$, the results for other lag order is similar. The $M_\varepsilon(m,l)$ tests are asymptotically one sided $N(0,1)$ test and upper-tailed critical values should be used, which are 1.65 and 2.33 at the 5% and 1% levels, respectively.